Pomůcka pro cvičení: 2. semestr Bc studia

Lokální extrémy funkcí dvou proměnných

Lokální extrémy

balíček: plots, LinearAlgebra

J. Steward: Calculus, str. 962

Příklad funkce dvou proměnných, která má dva stacionární body a v obou má lokální maximum.

$f(x,y) = -(x^2-1)^2 - (x^2y-x-1)^2$

POZOR: U funkce jedné proměnné tato situace nemůže nastat.

```
> with(plots):with(LinearAlgebra):
> f:=(x,y)->-(x^2-1)^2-(x^2*y-x-1)^2;
```

$$f := (x,y) \to -(x^2-1)^2 - (x^2y-x-1)^2$$

```
> diff(f(x,y),x);
```

$$-4(x^2-1)x - 2(x^2y-x-1)(2xy-1)$$

```
> diff(f(x,y),y);
```

$$-2(x^2y-x-1)x^2$$

```
> solve({,},{x,y});
```

$$\{x=1, y=2\}, \{x=-1, y=0\}$$

```
> fxx:=diff(f(x,y),x$2);
```

$$fxx := -12x^2 + 4 - 2(2xy-1)^2 - 4(x^2y-x-1)y$$

```
> fyy:=diff(f(x,y),y$2);
```

$$fyy := -2x^4$$

```
> fxy:=diff(f(x,y),x,y);
```

$$fxy := -2x^2(2xy-1) - 4(x^2y-x-1)x$$

```
> D2:=Matrix(2,2,[fxx,fxy,fxy,fyy]);
```

$$D2 := [[-12x^2 + 4 - 2(2xy-1)^2 - 4(x^2y-x-1)y, -2x^2(2xy - 1) - 4(x^2y-x-1)x], [-2x^2(2xy-1) - 4(x^2y-x-1)x, -2x^4]]$$

```
> Determinant(D2);
```

$$24x^6 - 40x^4 - 40x^6y^2 + 72x^5y + 56x^4y - 48x^3 - 16x^2$$

```
> x:=1:y:=2:Determinant(D2);
```

$$16$$

```
> x:=-1:y:=0:Determinant(D2);
```

$$16$$

```
> f(1,2);f(-1,0);
```

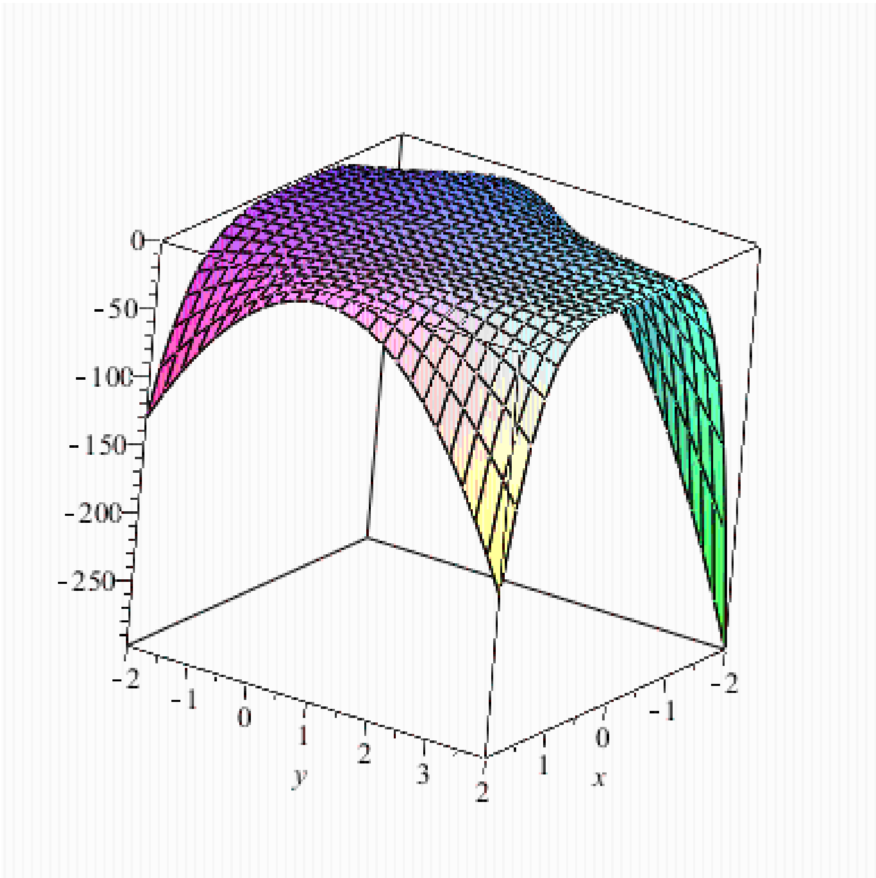
$$0$$

$$0$$

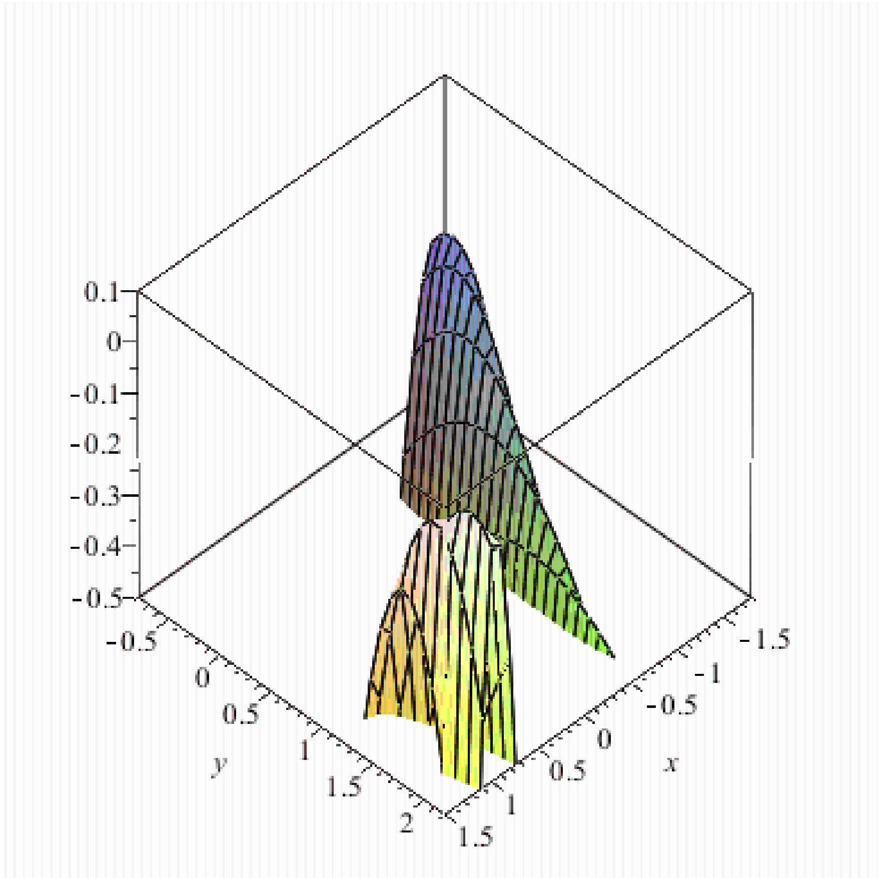
Graf funkce $f(x,y)$.

```
> restart;
```

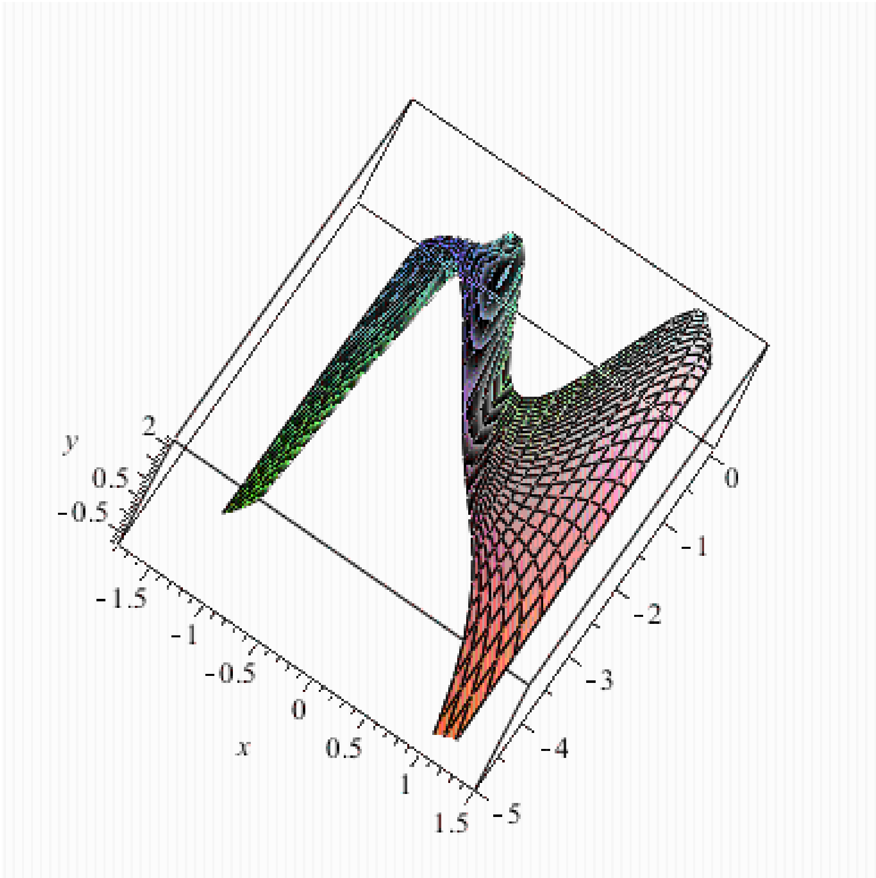
```
> with(plots):
> 
plot3d(-(x^2-1)^2-(x^2*y-x-1)^2,x=-2..2,y=-2..4,orientation=[46,6
9,13],axes=boxed);
```




```
> 
plot3d(-(x^2-1)^2-(x^2*y-x-1)^2,x=-1.2..1.2,y=-0.2..2.2,view=[-1.
8..1.5,-0.8..2.2,-0.5..0.1],axes=boxed);
```




```
>
plot3d(-(x^2-1)^2-(x^2*y-x-1)^2,x=-1.2..1.2,y=-0.2..2.2,view=[-1.
8..1.5,-0.8..2.2,-5..0.1],orientation=[-23,52,64],axes=boxed);
```



y
2
0.5
-0.5
-1.5
-1
-0.5
0
x
0.5
1
1.5
0
-1
-2
-3
-4
-5