

Pomůcka pro cvičení: 2. semestr Bc studia

Diferenciální rovnice 1. řádu

J. Kuben: Obyčejné diferenciální rovnice, ukázky řešení vybraných příkladů ze skript včetně zobrazení směrových polí a řešení počátečních podmínek.

Diferenciální rovnice 1. řádu

balíček: DEtools

V následujících příkladech najděte obecné řešení daných rovnic, pokud je zadána počáteční podmínka, najděte příslušné partikulární řešení. Partikulární řešení nakreslete.

Rovnice se separovanými proměnnými

Př. 1 $\frac{y}{y'} - x = 0, y(4) = 1, y(4) = 3.$

```
> restart;
```

```
> with(DEtools):
```

```
> DR:=y(x)/diff(y(x),x)-x=0;
```

$$DR := \frac{y(x)}{\frac{d}{dx}y(x)} - x = 0$$

```
> dsolve(DR,y(x));
```

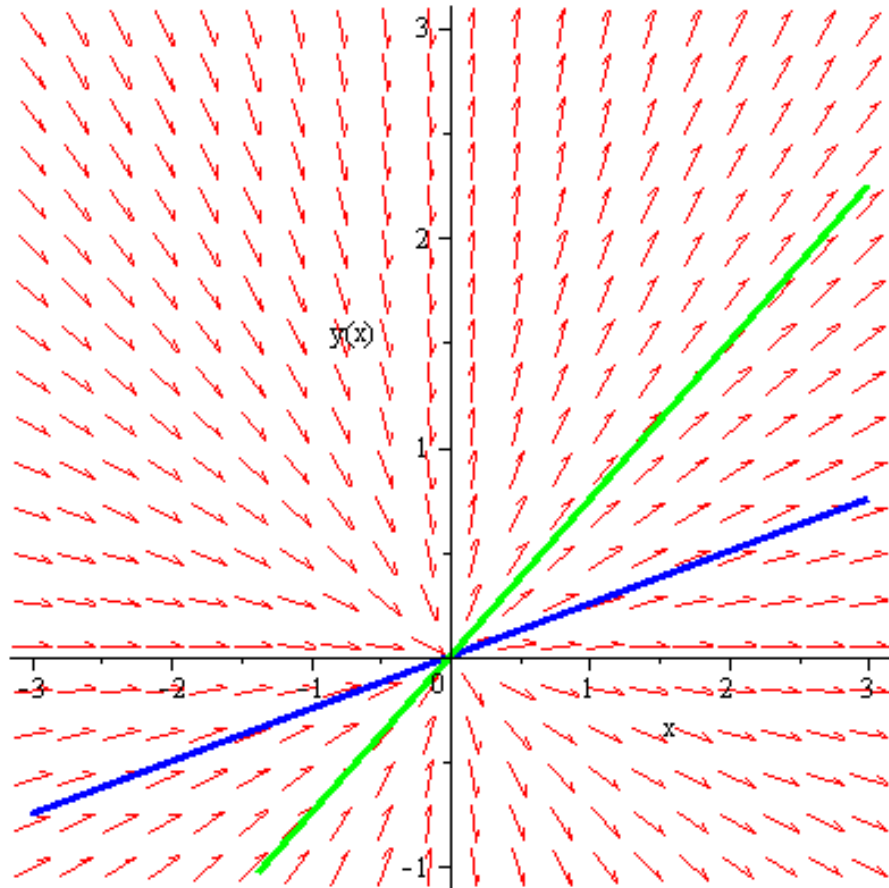
$$y(x) = _C1 x$$

```
> PP:=[y(4)=1,y(4)=3];
```

$$PP := [y(4) = 1, y(4) = 3]$$

```
>
```

```
DEplot([DR],[y(x)],x=-3..3,[[PP[1]],[PP[2]]],y=-1..3,linecolor=[blue,green]);
```



Př. 2 $xy^2 + x + yy' = 0, y(0) = 1, y(2) = 1.$

```
> restart;
> with(DEtools):
> DR:=x*(y(x))^2+x+y(x)*diff(y(x),x)-x^2*y(x)*diff(y(x),x)=0;
```

$$DR := xy(x)^2 + x + y(x) \left(\frac{d}{dx} y(x) \right) - x^2 y(x) \left(\frac{d}{dx} y(x) \right) = 0$$

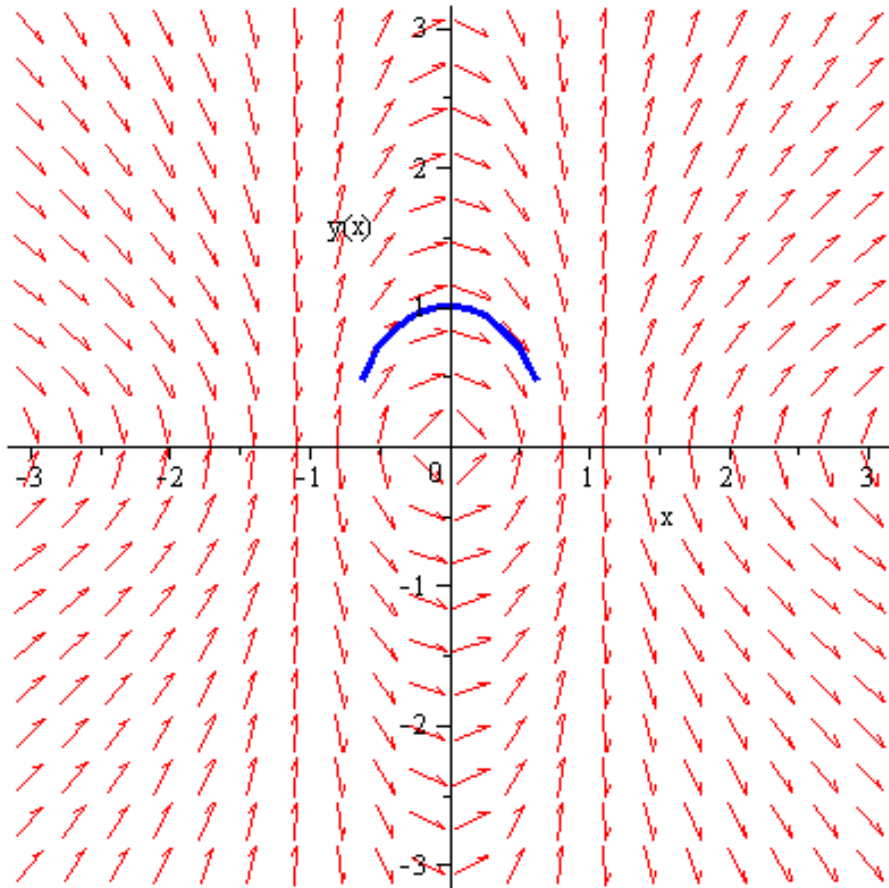
```
> dsolve(DR,y(x));
```

$$y(x) = \sqrt{-1 - _CI + _CI x^2}, y(x) = -\sqrt{-1 - _CI + _CI x^2}$$

```
>
```

```
DEplot([DR],[y(x)],x=-3..3,[[y(0)=1]],y=-3..3,linecolor=[blue]);
```

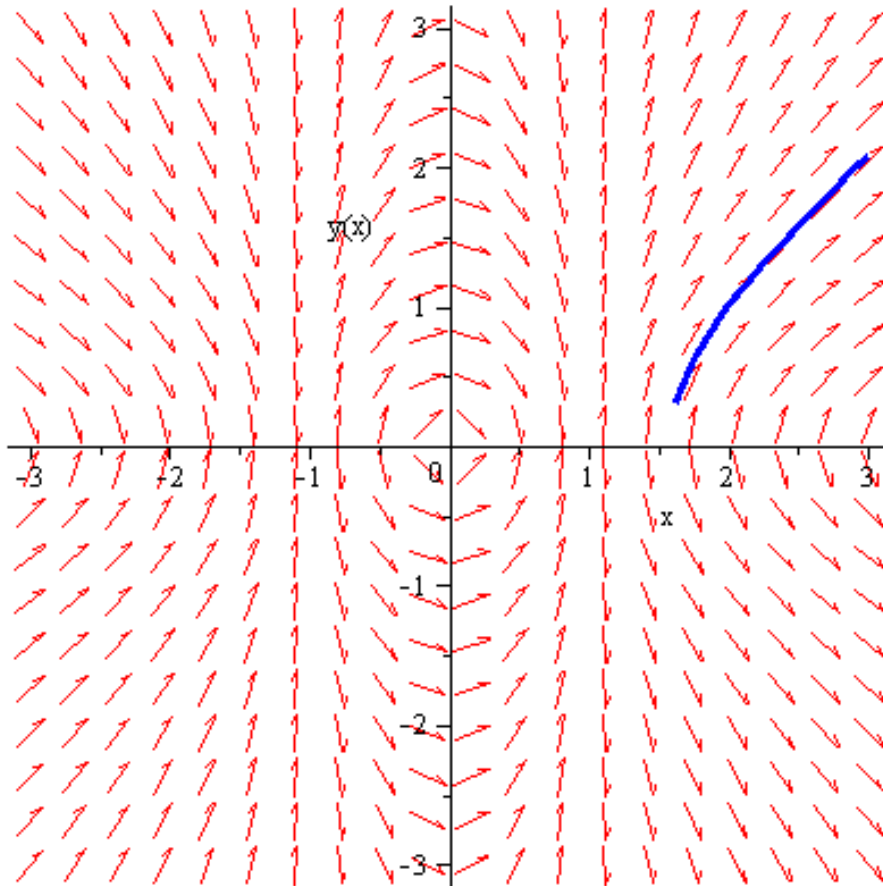
Warning, plot may be incomplete, the following error(s) were issued:
cannot evaluate the solution further right of .70710688, probably
a singularity
cannot evaluate the solution further left of -.70710688, probably
a singularity



>

```
DEplot([DR],[y(x)],x=-3..3,[[y(2)=1]],y=-3..3,linecolor=[blue]);
```

Warning, plot may be incomplete, the following error(s) were issued:
 cannot evaluate the solution further left of 1.5811388, probably
 a singularity



> restart;

Homogenní rovnice

Př. 1 $y' = \frac{x}{y} + \frac{y}{x}, y(1) = 2.1, y(-2) = 1.$

> restart;

> with(DEtools):

> DR:=diff(y(x),x)=x/y(x)+y(x)/x;

$$DR := \frac{d}{dx} y(x) = \frac{x}{y(x)} + \frac{y(x)}{x}$$

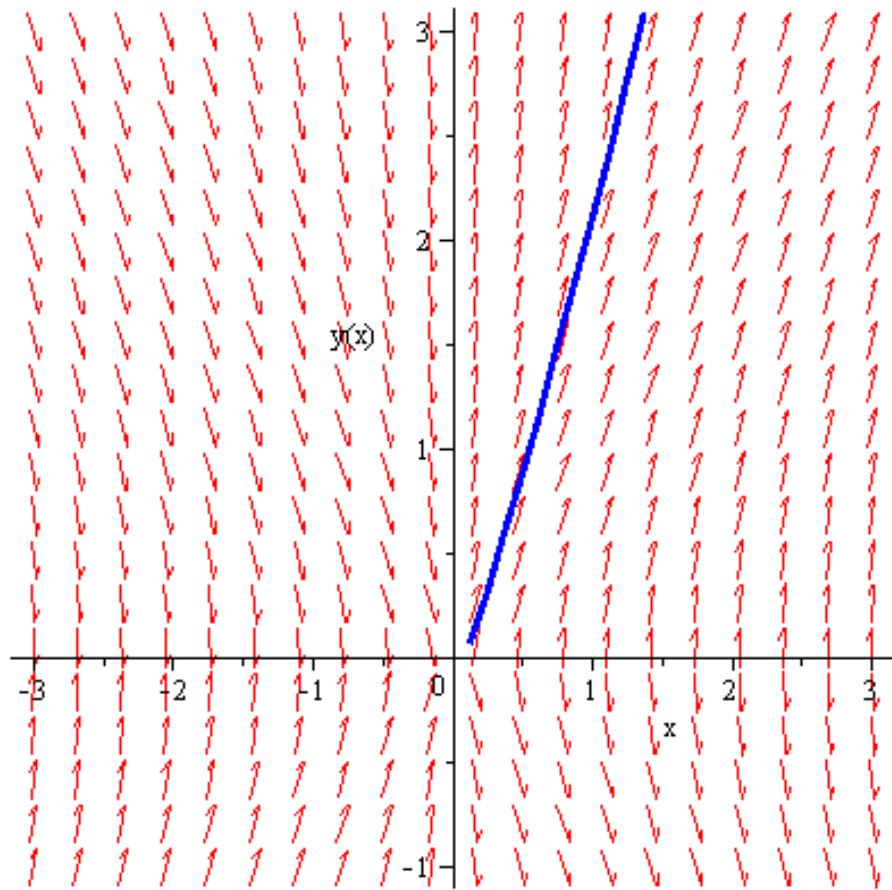
> dsolve(DR,y(x));

$$y(x) = \sqrt{2 \ln(x) + _C1} x, y(x) = -\sqrt{2 \ln(x) + _C1} x$$

>

DEplot([DR],[y(x)],x=-3..3,[[y(1)=2.1]],y=-1..3,linecolor=[blue])
;

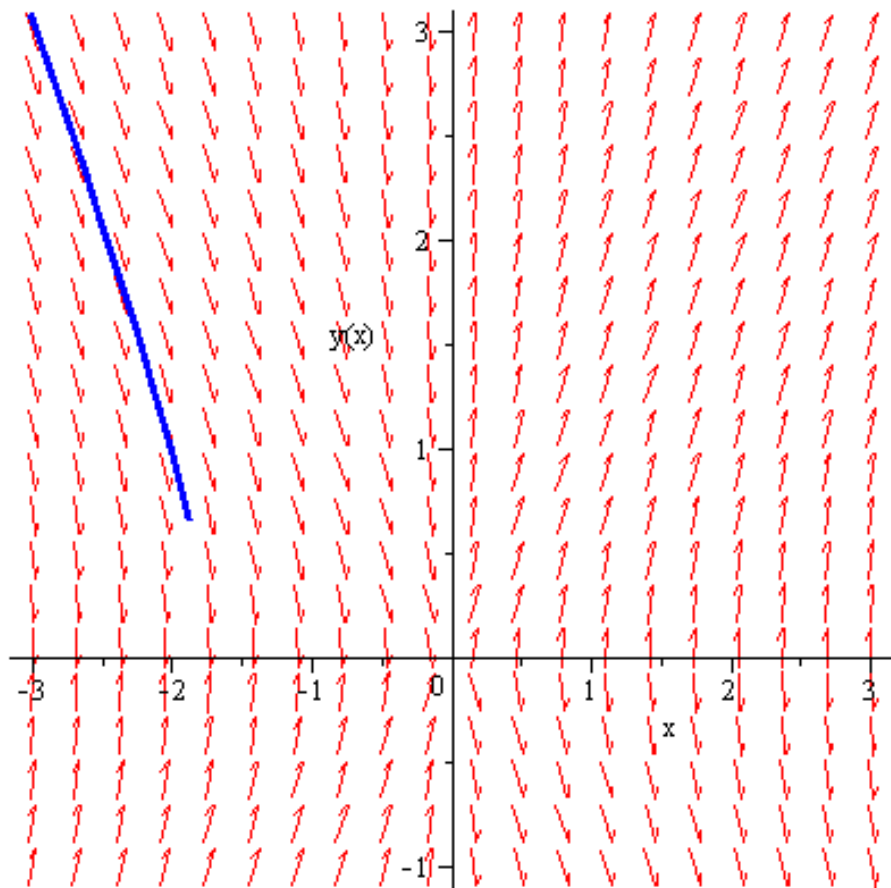
Warning, plot may be incomplete, the following error(s) were issued:
cannot evaluate the solution further left of .11025057, probably
a singularity



>

```
DEplot([DR],[y(x)],x=-3..3,[[y(-2)=1]],y=-1..3,linecolor=[blue]);
```

Warning, plot may be incomplete, the following error(s) were issued:
cannot evaluate the solution further right of -1.7649938, probably
a singularity



Př. 2 $y' = \cos(x - y)$, $y\left(\frac{\pi}{2}\right) = 1$.

```
> restart;
```

```
> with(DEtools):
```

```
> DR:=diff(y(x),x)=cos(x-y(x));
```

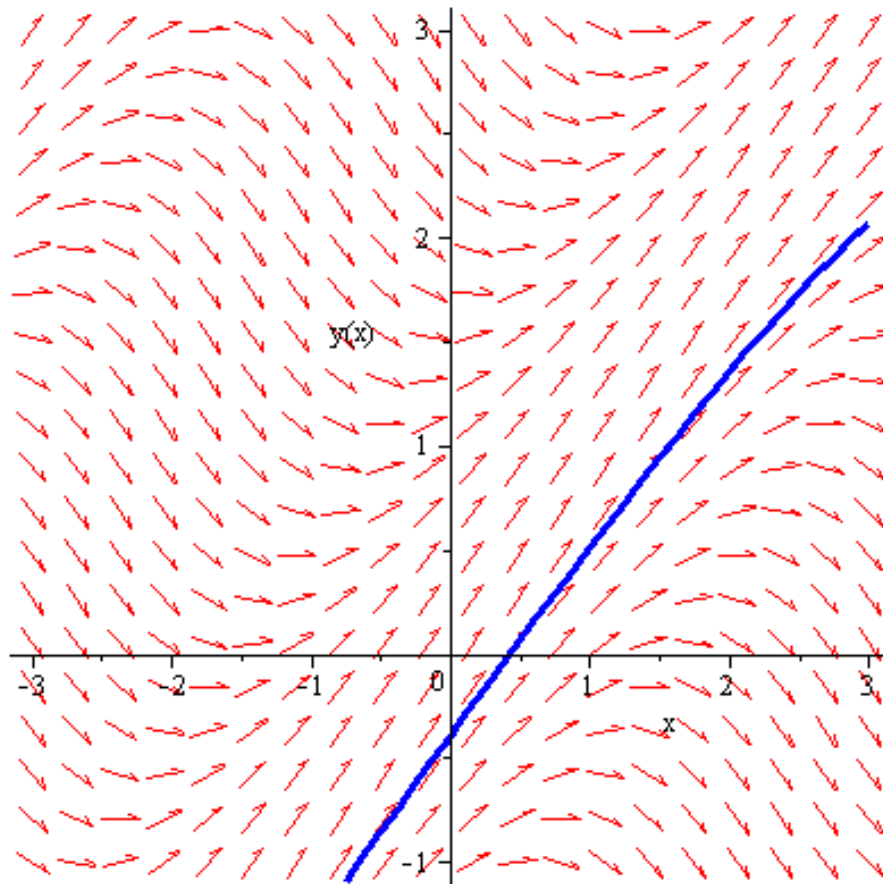
$$DR := \frac{d}{dx} y(x) = \cos(x - y(x))$$

```
> dsolve(DR,y(x));
```

$$y(x) = x - 2 \arctan\left(\frac{1}{-x + _CI}\right)$$

```
>
```

```
DEplot([DR],[y(x)],x=-3..3,[[y(Pi/2)=1]],y=-1..3,linecolor=[blue]) ;
```



>

Lineární diferenciální rovnice

Př. 1 $y' - y \operatorname{tg}(x) = \frac{1}{\cos(x)}$, PP $y(0) = 0$.

> restart;

> with(DEtools):

> DR:=diff(y(x),x)-y(x)*tan(x)=1/cos(x);

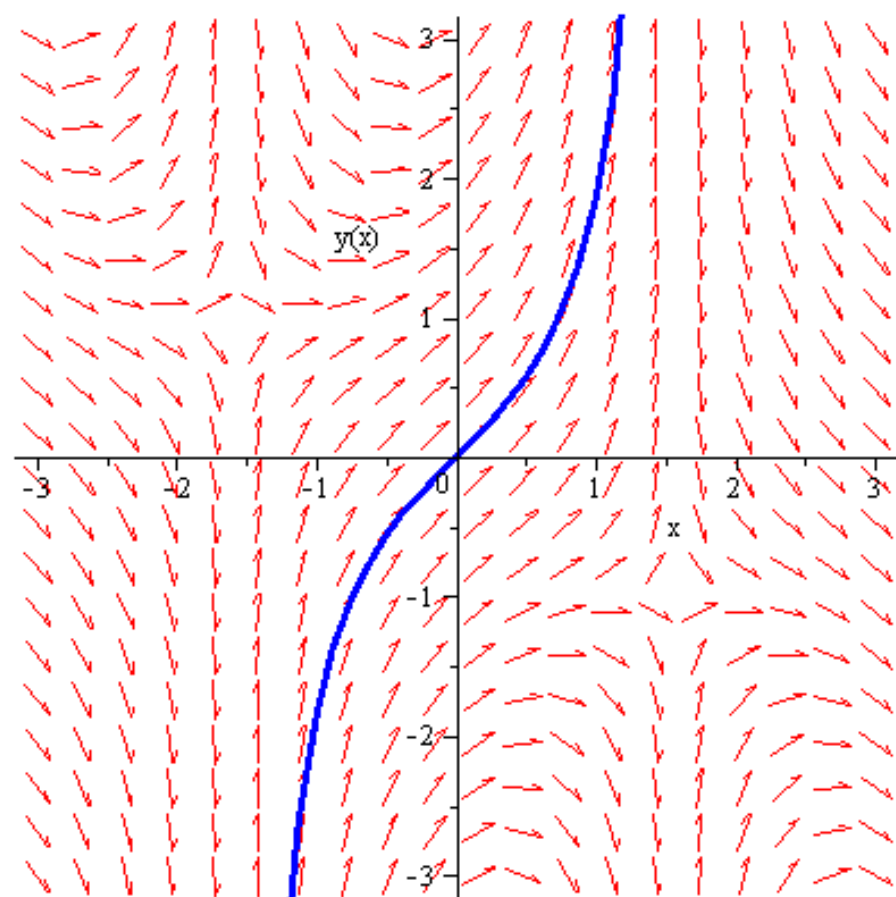
$$DR := \frac{d}{dx} y(x) - y(x) \tan(x) = \frac{1}{\cos(x)}$$

> dsolve(DR,y(x));

$$y(x) = \frac{x + CI}{\cos(x)}$$

>

DEplot([DR],[y(x)],x=-3..3,[y(0)=0],y=-3..3,linecolor=[blue]);



>