

Pomůcka pro cvičení: 3. semestr Bc studia

Číselné řady

balíček: `plots`, `ListTools`

Př. 1 Je dána řada $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$.

a) Určete s_{10} .

b) Napište prvních deset členů řady, členy graficky zobrazte.

c) Graficky zobrazte prvních deset částečných součtů.

d) Určete součet řady

```
> restart:
```

```
> with (plots) : with (ListTools) :
```

ad a)

```
> Sum ( (2^n+3^n) / 5^n, n=1..10) = sum ( (2^n+3^n) / 5^n, n=1..10) ;
```

$$\sum_{n=1}^{10} \frac{2^n + 3^n}{5^n} = \frac{21069598}{9765625}$$

ad b)

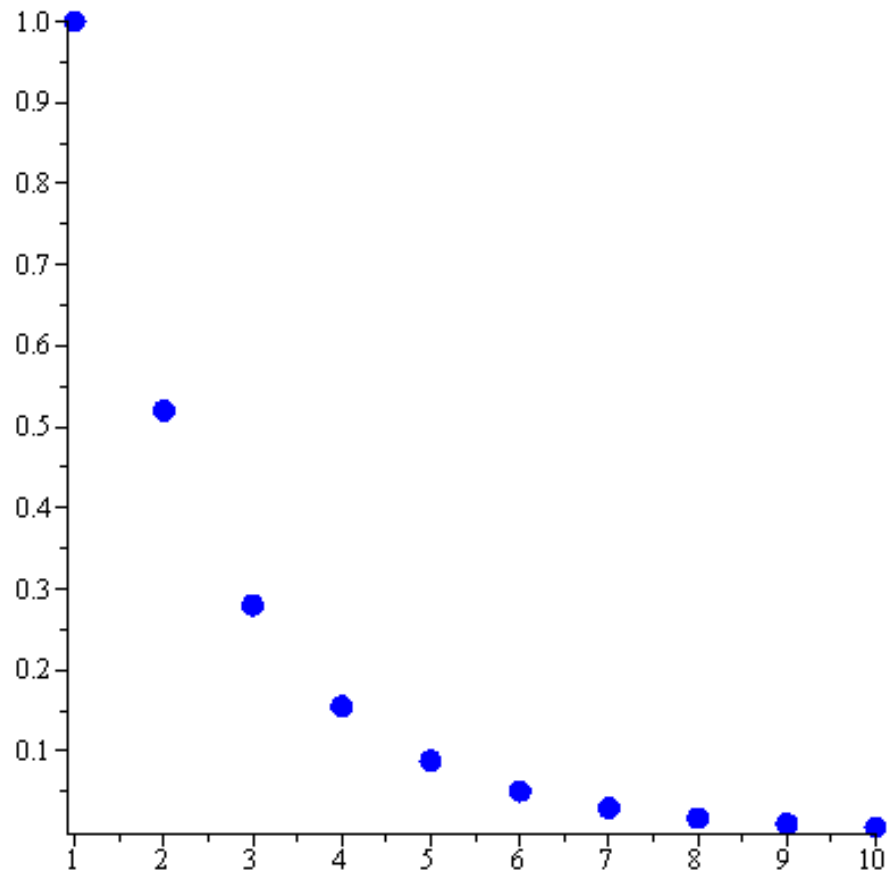
```
> [seq ( (2^n+3^n) / 5^n, n=1..10) ] ;
```

$$\left[1, \frac{13}{25}, \frac{7}{25}, \frac{97}{625}, \frac{11}{125}, \frac{793}{15625}, \frac{463}{15625}, \frac{6817}{390625}, \frac{4039}{390625}, \frac{60073}{9765625} \right]$$

```
> data := [seq ( [n, (2^n+3^n) / 5^n], n=1..10) ] ;
```

$$data := \left[\left[1, 1 \right], \left[2, \frac{13}{25} \right], \left[3, \frac{7}{25} \right], \left[4, \frac{97}{625} \right], \left[5, \frac{11}{125} \right], \left[6, \frac{793}{15625} \right], \right. \\ \left. \left[7, \frac{463}{15625} \right], \left[8, \frac{6817}{390625} \right], \left[9, \frac{4039}{390625} \right], \left[10, \frac{60073}{9765625} \right] \right]$$

```
> pointplot (data, symbol=solidcircle, color=blue, symbolsize=20) ;
```



ad c)

```
> L:=ListTools['PartialSums']( );
```

$$L := \left[1, \frac{38}{25}, \frac{9}{5}, \frac{1222}{625}, \frac{1277}{625}, \frac{32718}{15625}, \frac{33181}{15625}, \frac{836342}{390625}, \frac{840381}{390625}, \frac{21069598}{9765625} \right]$$

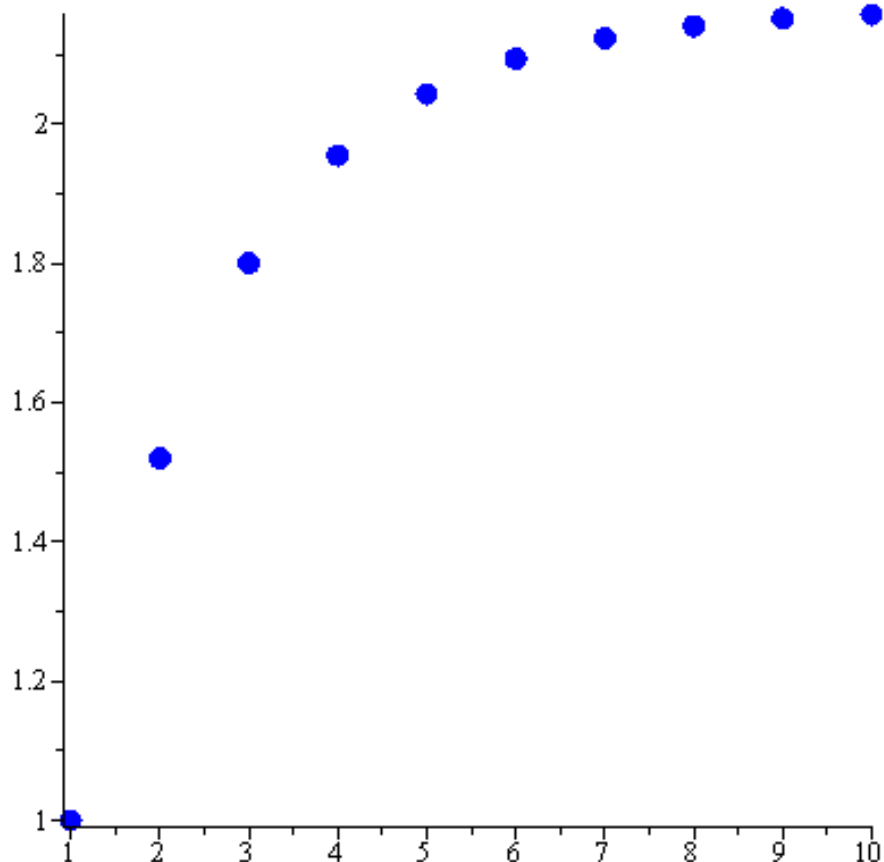
```
> evalf[5]( );
```

$$[1., 1.5200, 1.8000, 1.9552, 2.0432, 2.0940, 2.1236, 2.1410, 2.1514, 2.1575]$$

```
> data1:=Enumerate(L);
```

$$data1 := \left[[1, 1], \left[2, \frac{38}{25} \right], \left[3, \frac{9}{5} \right], \left[4, \frac{1222}{625} \right], \left[5, \frac{1277}{625} \right], \left[6, \frac{32718}{15625} \right], \left[7, \frac{33181}{15625} \right], \left[8, \frac{836342}{390625} \right], \left[9, \frac{840381}{390625} \right], \left[10, \frac{21069598}{9765625} \right] \right]$$

```
> pointplot(data1, symbol=solidcircle, color=blue, symbolsize=20);
```



ad d)

```
>
Sum((2^n+3^n)/5^n,n=1..infinity)=sum((2^n+3^n)/5^n,n=1..infinity)
;
```

$$\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n} = \frac{13}{6}$$

Př. 2 Aproximujte součet alternující řady $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{n!}$ součtem jejích prvních šesti členů.

```
> restart:
> with(plots):with(ListTools):
> Sum((-1)^(n-1)*1/n!,n=1..6);
```

$$\sum_{n=1}^6 \frac{(-1)^{n-1}}{n!}$$

Pro lepší představu si prvních 6 členů řady znázorníme graficky.

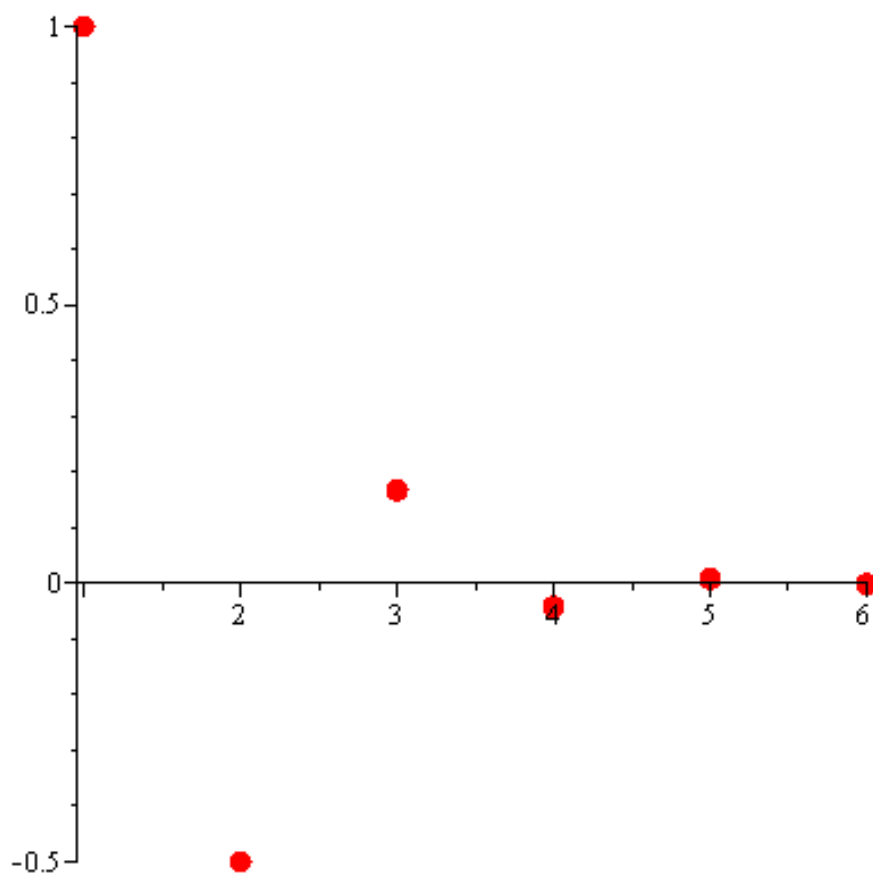
```
> S:= [seq((-1)^(n-1)*1/n!,n=1..6)];
```

$$S := \left[1, -\frac{1}{2}, \frac{1}{6}, -\frac{1}{24}, \frac{1}{120}, -\frac{1}{720} \right]$$

```
> data:= [seq([n, (-1)^(n-1)*1/n!],n=1..6)];
```

$$data := \left[[1, 1], \left[2, -\frac{1}{2} \right], \left[3, \frac{1}{6} \right], \left[4, -\frac{1}{24} \right], \left[5, \frac{1}{120} \right], \left[6, -\frac{1}{720} \right] \right]$$

```
> pointplot(data,symbol=solidcircle,color=red,symbolsize=20);
```



Dále vytvoříme prvních 6 částečných součtů, které také zobrazíme.

```
> ListTools['PartialSums']( );
```

$$\left[1, \frac{1}{2}, \frac{2}{3}, \frac{5}{8}, \frac{19}{30}, \frac{91}{144}\right]$$

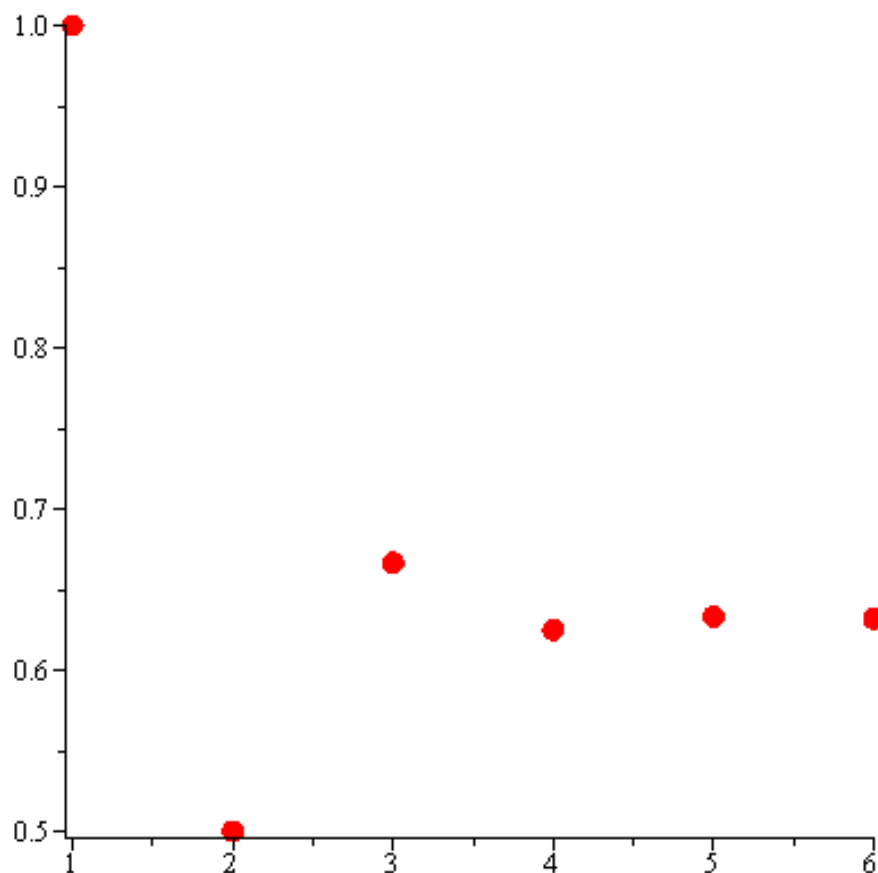
```
> L:=evalf[5]( );
```

$$L := [1., 0.50000, 0.66667, 0.62500, 0.63333, 0.63194]$$

```
> data1:=Enumerate(L);
```

$$data1 := [[1, 1.], [2, 0.50000], [3, 0.66667], [4, 0.62500], [5, 0.63333], [6, 0.63194]]$$

```
> pointplot(data1,symbol=solidcircle,color=red,symbolsize=20);
```



```
> Sum((-1)^(n-1)*1/n!, n=1..6)=sum((-1)^(n-1)*1/n!, n=1..6);
```

$$\sum_{n=1}^6 \frac{(-1)^{n-1}}{n!} = \frac{91}{144}$$

Grandiho řada

balíček: plots, ListTools

```
> restart;
```

```
> with(plots):with(ListTools):
```

```
> Sum((-1)^(n-1), n=1..infinity);
```

$$\sum_{n=1}^{\infty} (-1)^{n-1}$$

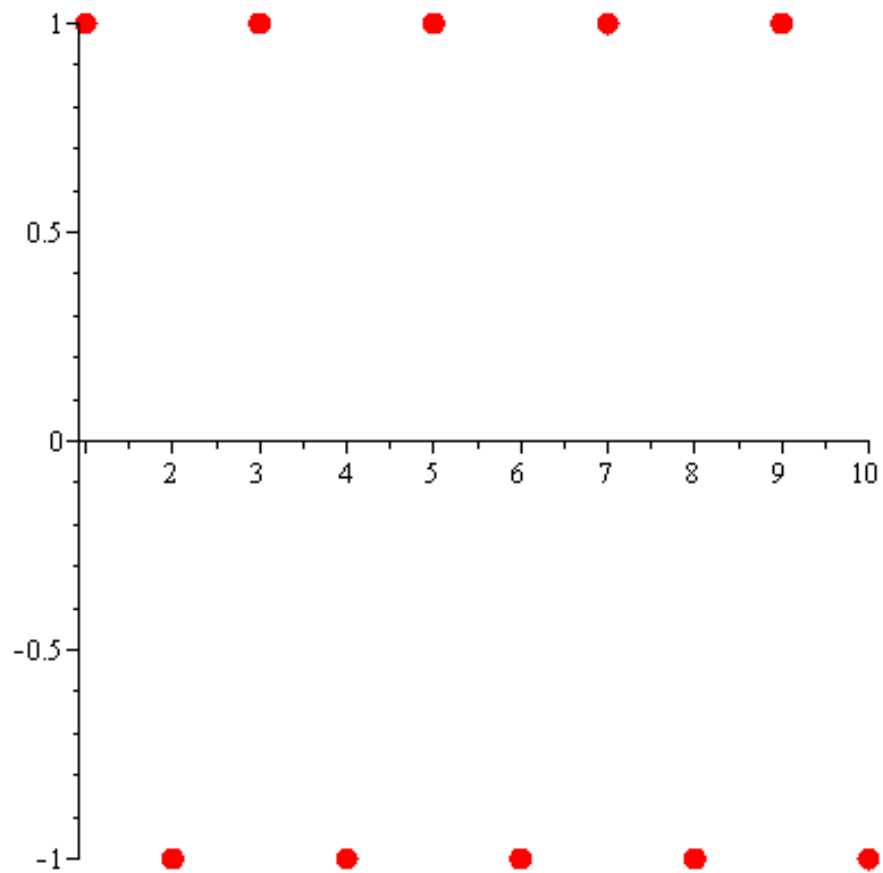
```
> S:= [seq((-1)^(n-1), n=1..10)];
```

```
S:= [1, -1, 1, -1, 1, -1, 1, -1, 1, -1]
```

```
> data:=Enumerate(S);
```

```
data:= [[1, 1], [2, -1], [3, 1], [4, -1], [5, 1], [6, -1], [7, 1], [8, -1],
[9, 1], [10, -1]]
```

```
> pointplot(data, symbol=solidcircle, color=red, symbolsize=20);
```



Částečné součty Grandiho řady.

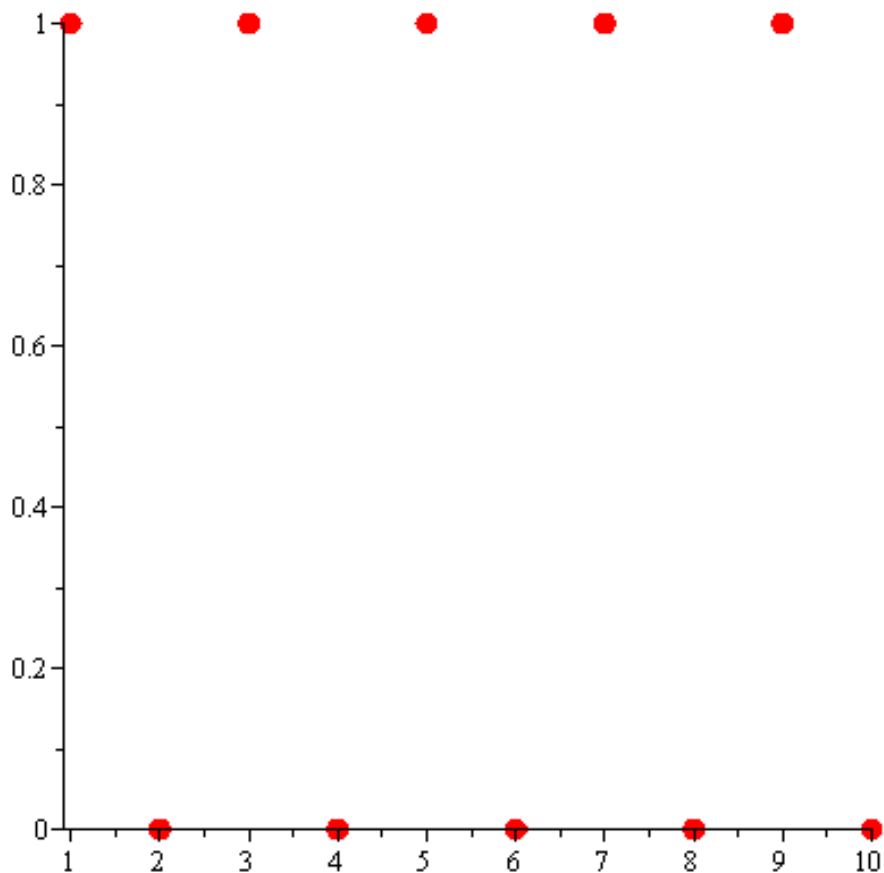
```
> L:=ListTools['PartialSums']( );
```

```
L := [1, 0, 1, 0, 1, 0, 1, 0, 1, 0]
```

```
> data1:=Enumerate(L);
```

```
data1 := [[1, 1], [2, 0], [3, 1], [4, 0], [5, 1], [6, 0], [7, 1], [8, 0], [9, 1],  
[10, 0]]
```

```
> pointplot(data1, symbol=solidcircle, color=red, symbolsize=20);
```



```
> assume(n>1):
> Sum((-1)^(n-1),n=1..infinity)=sum((-1)^(n-1),n=1..infinity);
```

$$\sum_{n=1}^{\infty} (-1)^{n-1} = \sum_{n=1}^{\infty} (-1)^{n-1}$$

Řada nemá součet = diverguje.

```
>
```

Harmonická řada

balíček: plots, ListTools

```
> restart;
> with(plots):with(ListTools):
> Sum(1/n,n=1..infinity);
```

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

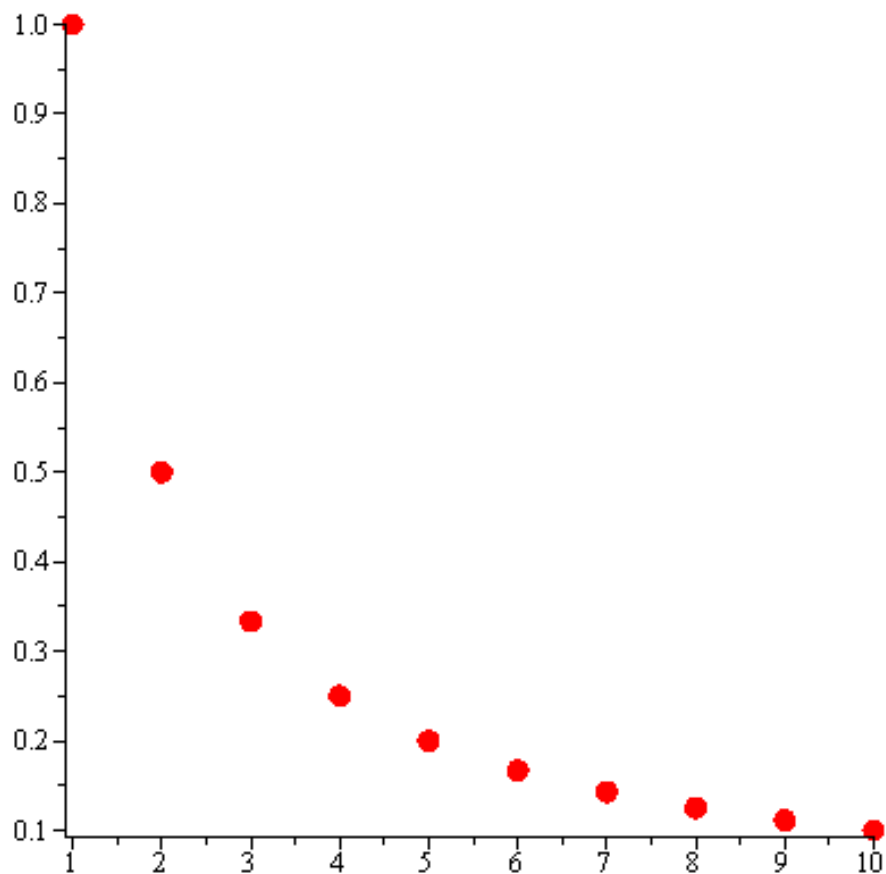
```
> S:= [seq(1/n,n=1..10)];
```

$$S := \left[1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10} \right]$$

```
> data:=Enumerate(S);
```

```
data := [ [1, 1], [2,  $\frac{1}{2}$ ], [3,  $\frac{1}{3}$ ], [4,  $\frac{1}{4}$ ], [5,  $\frac{1}{5}$ ], [6,  $\frac{1}{6}$ ], [7,  $\frac{1}{7}$ ], [8,  $\frac{1}{8}$ ], [9,  $\frac{1}{9}$ ], [10,  $\frac{1}{10}$  ] ]
```

```
> pointplot(data, symbol=solidcircle, color=red, symbolsize=20);
```



Částečné součty harmonické řady.

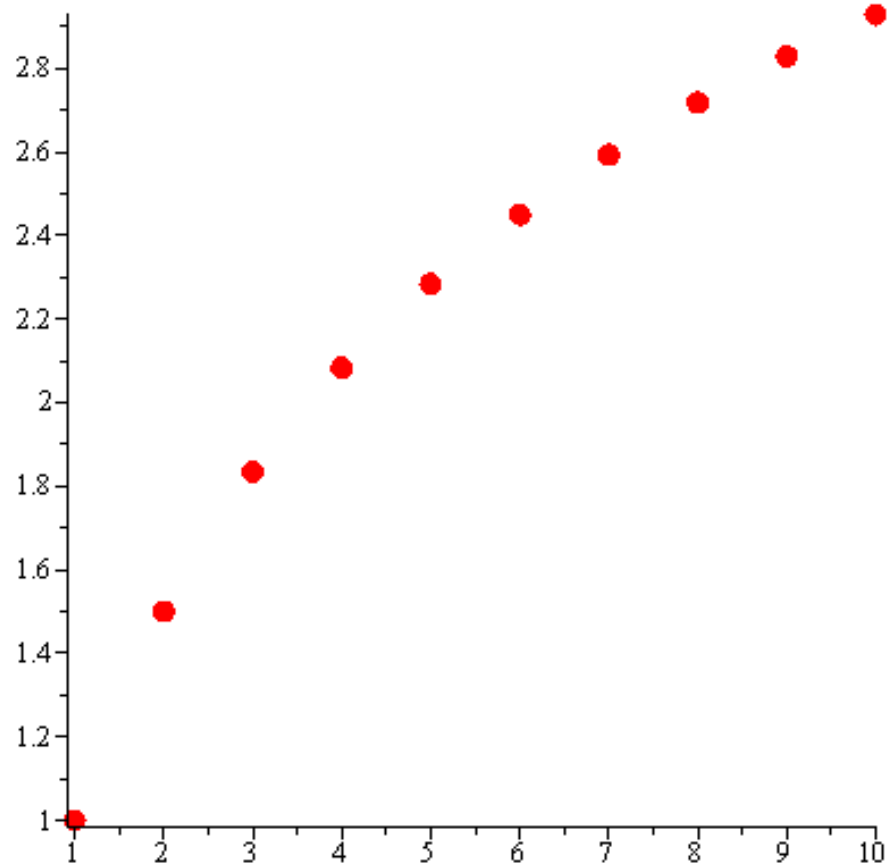
```
> L:=ListTools['PartialSums']( );
```

```
L := [ 1,  $\frac{3}{2}$ ,  $\frac{11}{6}$ ,  $\frac{25}{12}$ ,  $\frac{137}{60}$ ,  $\frac{49}{20}$ ,  $\frac{363}{140}$ ,  $\frac{761}{280}$ ,  $\frac{7129}{2520}$ ,  $\frac{7381}{2520}$  ]
```

```
> data1:=Enumerate(L);
```

```
data1 := [ [1, 1], [2,  $\frac{3}{2}$ ], [3,  $\frac{11}{6}$ ], [4,  $\frac{25}{12}$ ], [5,  $\frac{137}{60}$ ], [6,  $\frac{49}{20}$ ], [7,  $\frac{363}{140}$ ], [8,  $\frac{761}{280}$ ], [9,  $\frac{7129}{2520}$ ], [10,  $\frac{7381}{2520}$  ] ]
```

```
> pointplot(data1, symbol=solidcircle, color=red, symbolsize=20);
```

```
> Sum(1/n,n=1..infinity)=sum(1/n,n=1..infinity) ;
```

$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

Řada nemá součet = diverguje.