Pomůcka pro přednášku: 3. semestr Bc studia

Generování náhodných výběrů z různých typů rozdělení

```
> with(Statistics):
> infolevel[Statistics]:=1;
```

$$infolevel_{Statistics} := 1$$

```
> with(RandomTools[MersenneTwister]):
> SetState(state=249357846);
```

Generování náhodného výběru při házení regulérní kostkou

```
> P:=[seq(1/6,i=1..6)];
```

$$P := \left[\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right]$$

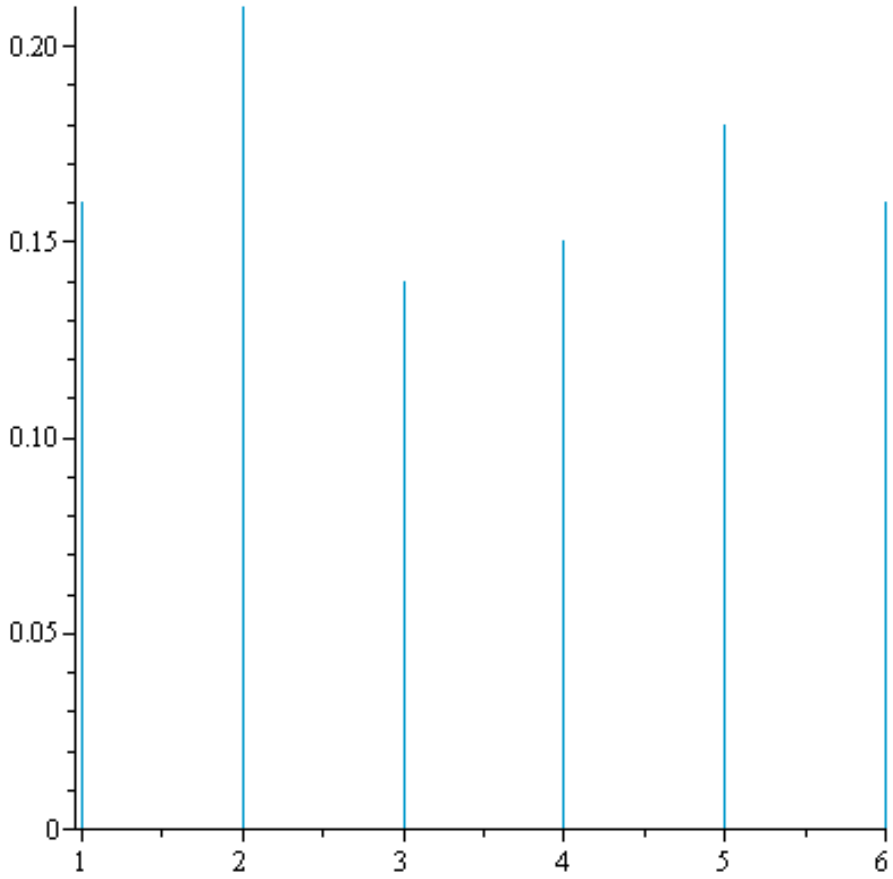
```
> X:=RandomVariable(ProbabilityTable(P));
```

$$X := \_R21$$

```
> Data:=Sample(X,100);
```

$$Data := \begin{bmatrix} 1 .. 100\ Vector_{row} \\ Data\ Type: integer_4 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

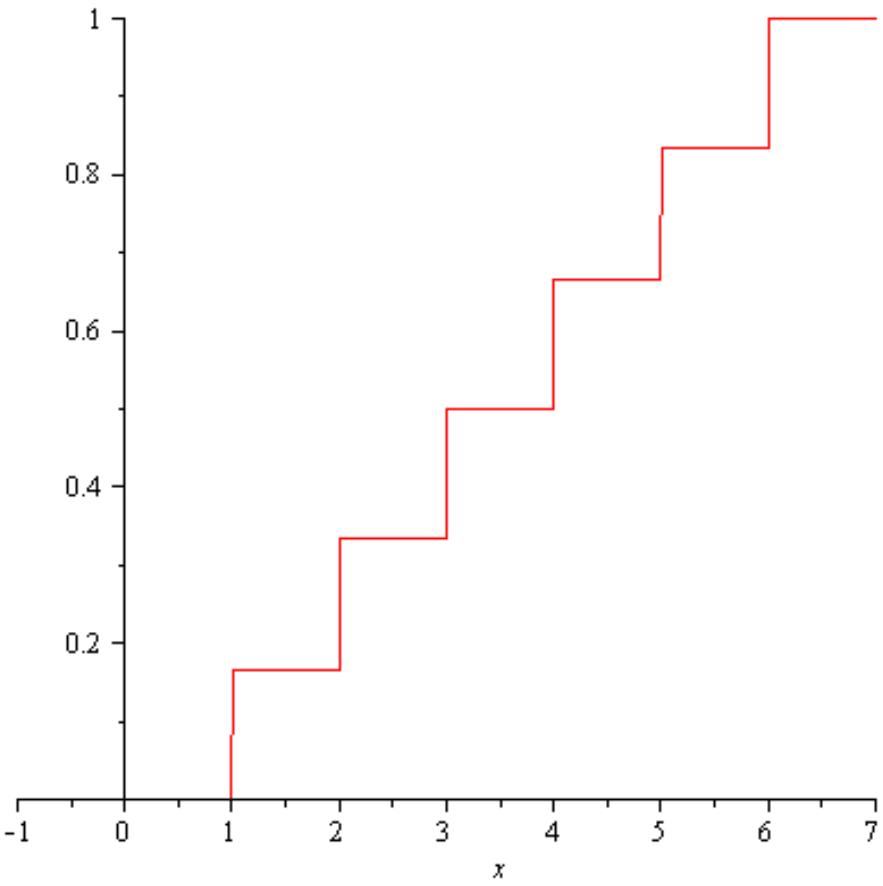
```
> Histogram(Data,discrete=true);
Histogram Type:  discrete
Data Range:      1. .. 6.
Number of Bins:  6
Frequency Scale: relative
```




```
> CDF(X,x);
```

$$\sum_{k=1}^{\max(0,\ \min(6,\ \mathrm{floor}(x)))} \left[\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right]_k$$

```
> plot(,x=-1..7,discont=true);
```



Generování náhodného výběru z Poissonova rozdělení, střední počet událostí je 3,5.

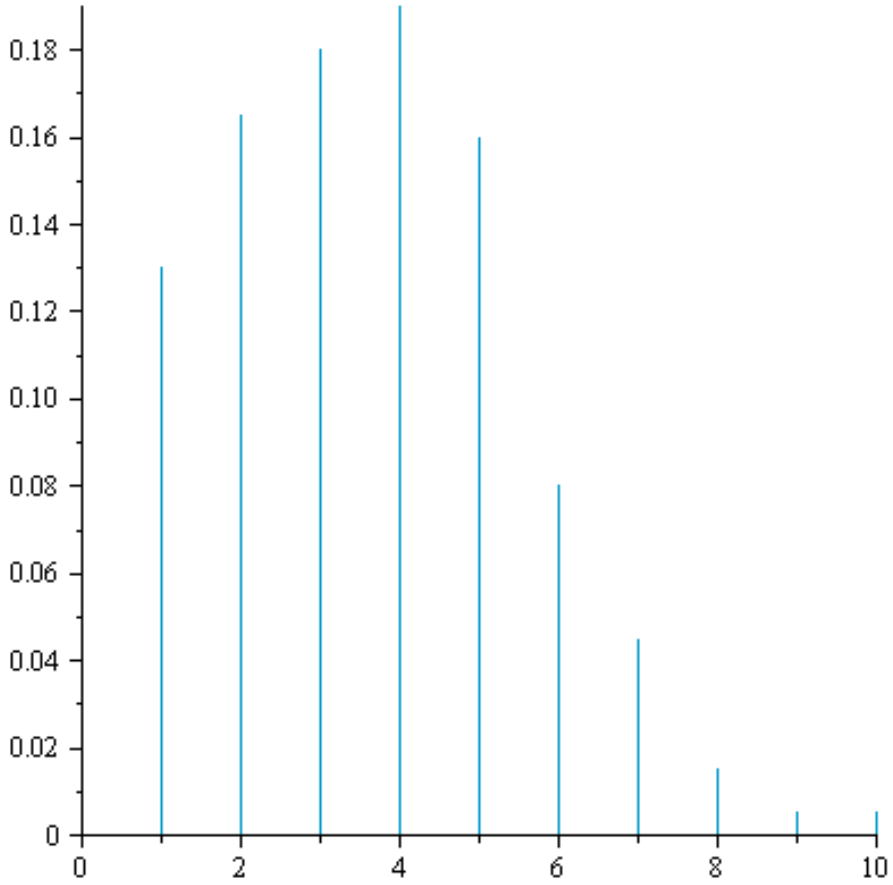
```
> Y:=RandomVariable(Poisson(3.5));
```

$$Y := \_R22$$

```
> Data1:=Sample(Y,200);
```

$$Data1 := \begin{bmatrix} 1 .. 200\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
> Histogram(Data1,discrete=true);
Histogram Type:  discrete
Data Range:      0. .. 10.
Number of Bins:  11
Frequency Scale: relative
```




```
> ProbabilityFunction(Poisson(lambda),k);
ProbabilityFunction(Y,k);
```

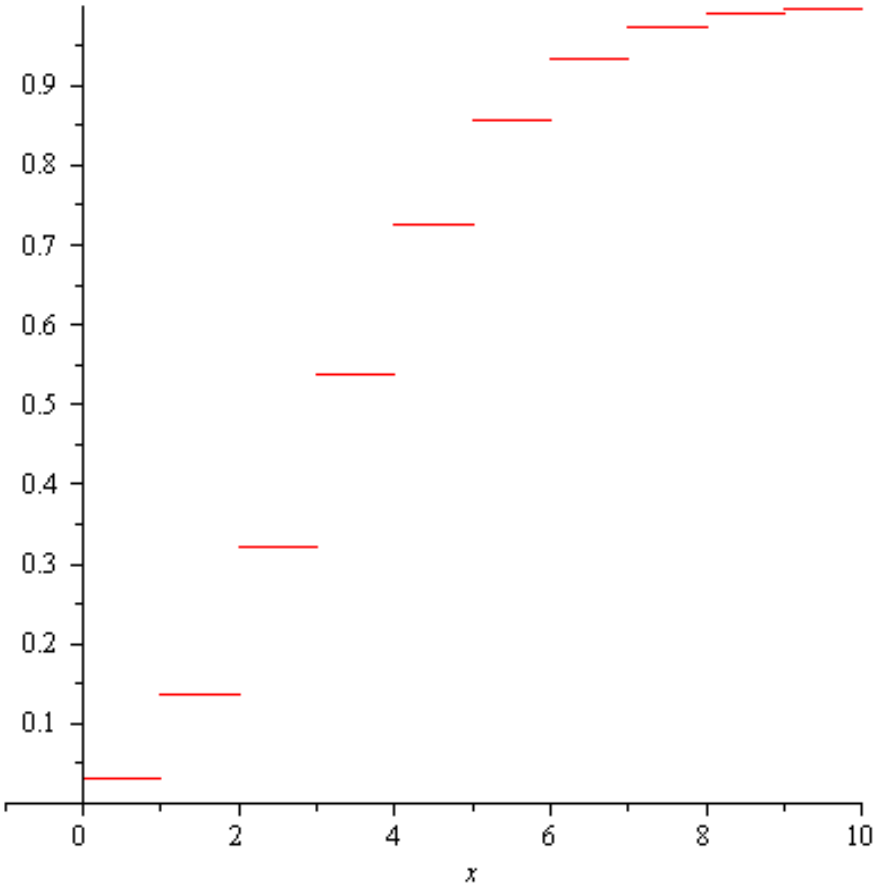
$$\begin{cases} 0 & k < 0 \\ \dfrac{\lambda^{k}\, e^{-\lambda}}{k!} & otherwise \end{cases}$$

$$\begin{cases} 0 & k < 0 \\ \dfrac{0.030197383423.5^{k}}{k!} & otherwise \end{cases}$$

```
> CDF(Y,x);
```

$$\frac{1.000000000\,\Gamma(\max(-1., \mathrm{floor}(x)) + 1., 3.500000000)}{\Gamma(\max(-1., \mathrm{floor}(x)) + 1.)}$$

```
> plot(,x=-1..10,discont=true);
```



Generování náhodného výběru z normálního rozdělení se střední hodnotou -2 a směrodatnou odchylkou 1,69.

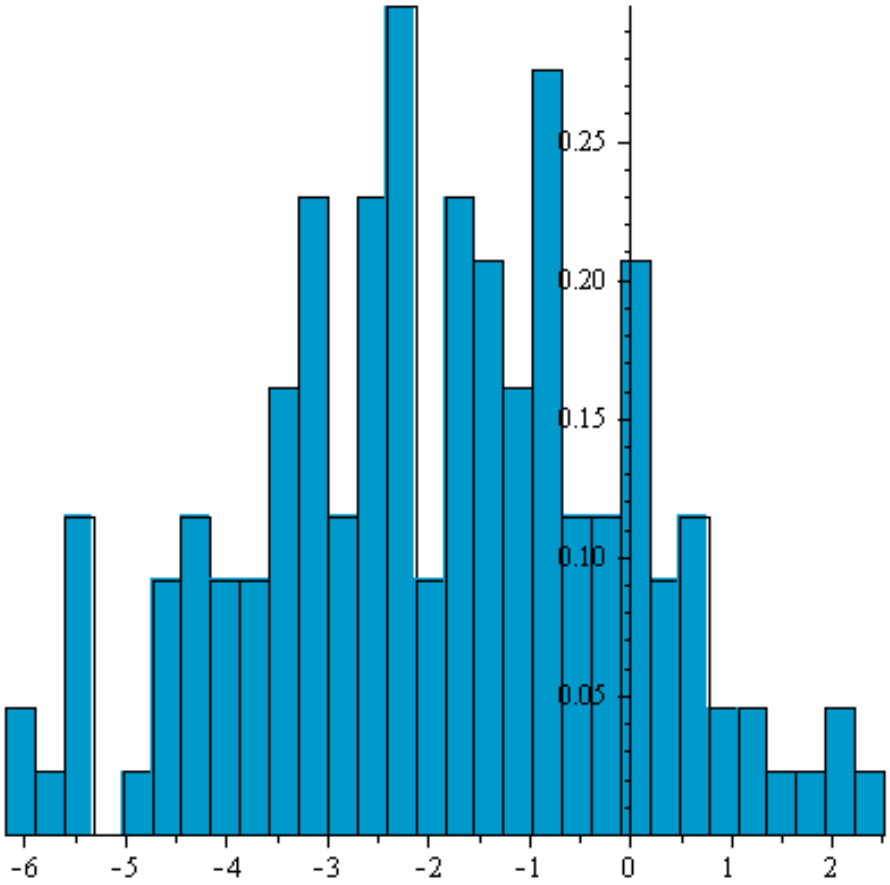
```
> Z:=RandomVariable(Normal(-2,1.69));
```

$$Z := \_R24$$

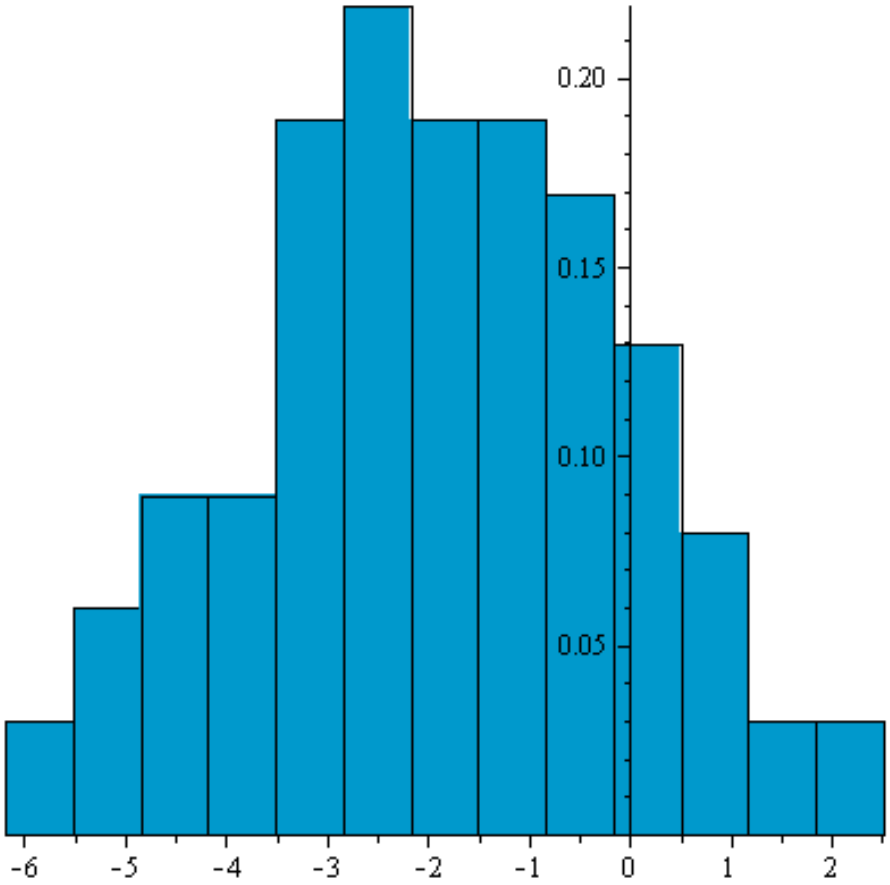
```
> Data2:=Sample(Z,150);
```

$$Data2 := \begin{bmatrix} 1 .. 150\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

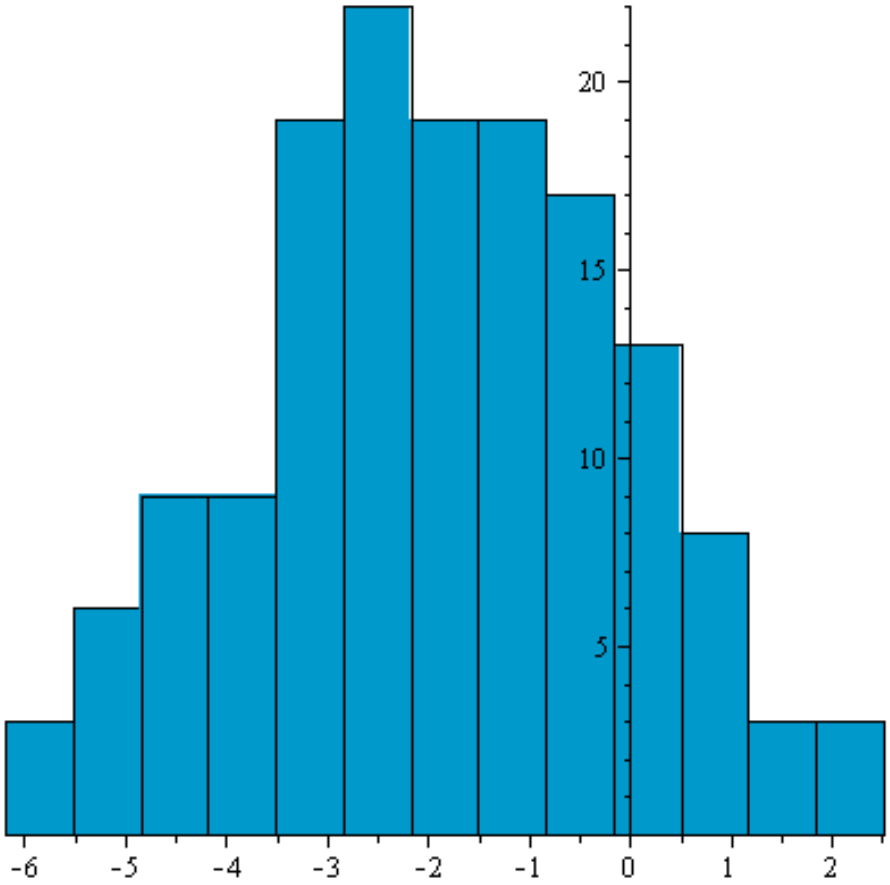
```
> Histogram(Data2);
Histogram Type:  default
Data Range:      -6.190827411 .. 2.512343994
Bin Width:       .2901057135
Number of Bins:  30
Frequency Scale: relative
```




```
> Histogram(Data2,bincount=13);
Histogram Type:  default
Data Range:      -6.190827411 .. 2.512343994
Bin Width:       .6694747235
Number of Bins:  13
Frequency Scale: relative
```




```
> Histogram(Data2,bincount=13,frequencyscale=absolute);
Histogram Type:  default
Data Range:      -6.190827411 .. 2.512343994
Bin Width:       .6694747235
Number of Bins:  13
Frequency Scale: absolute
```



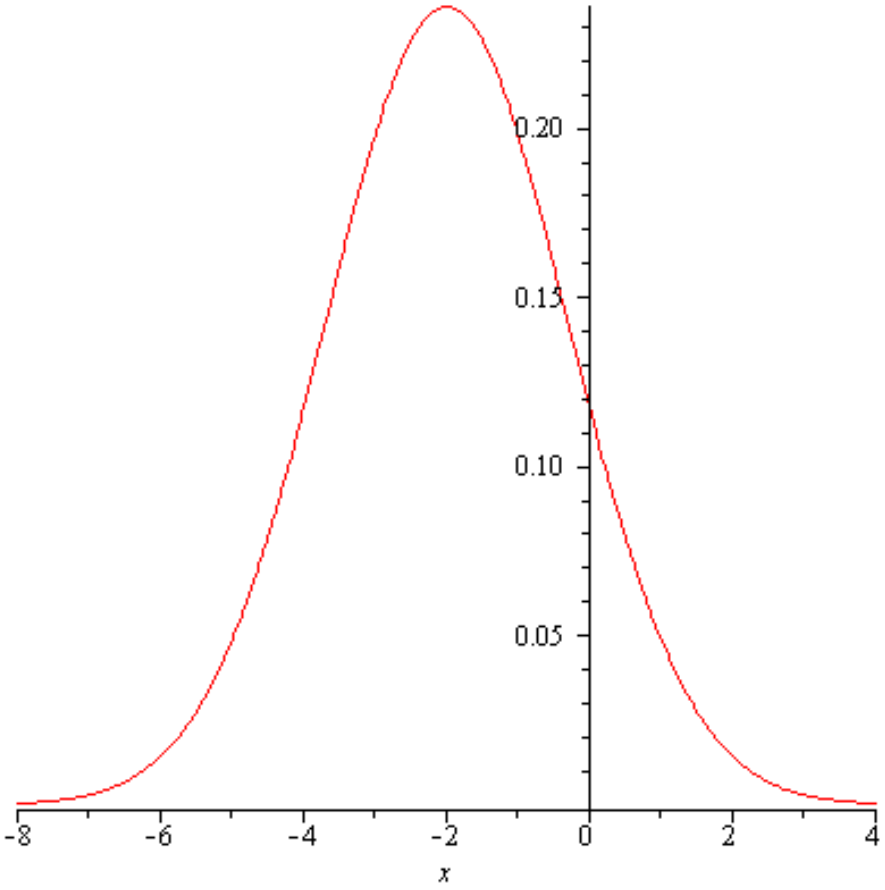

```
> PDF(Normal(mu,sigma),x);
```

$$\frac{\frac{1}{2}\,\sqrt{2}\,e^{-\frac{1}{2}\,\frac{(x-\mu)^2}{\sigma^2}}}{\sqrt{\pi}\,\sigma}$$

```
> PDF(Z,x);
```

$$\frac{0.2958579882\,\sqrt{2}\,e^{-0.1750638983\,(x+2)^2}}{\sqrt{\pi}}$$

```
> plot(,x=-8..4);
```



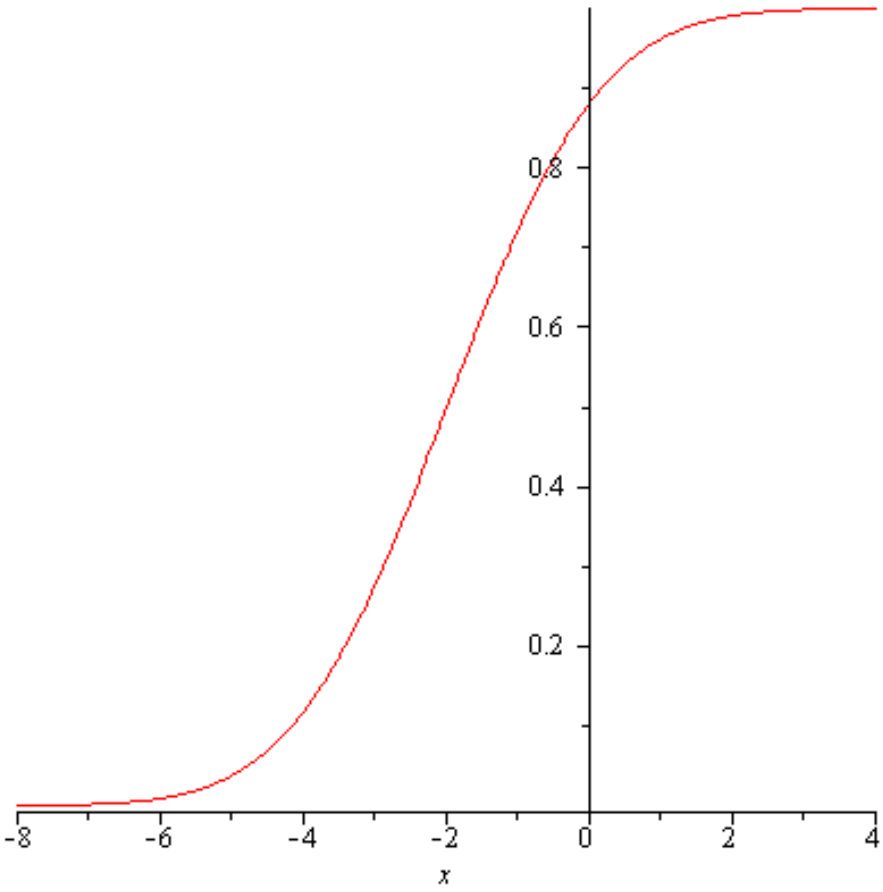

```
> CDF(Normal(mu,sigma),x);
```

$$\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(\frac{1}{2}\,\frac{(x-\mu)\sqrt{2}}{\sigma}\right)$$

```
> CDF(Z,x);
```

$$\frac{1}{2}+\frac{1}{2}\,\mathrm{erf}\left(0.2958579882\,(x+2)\sqrt{2}\right)$$

```
> plot(,x=-8..4);
```



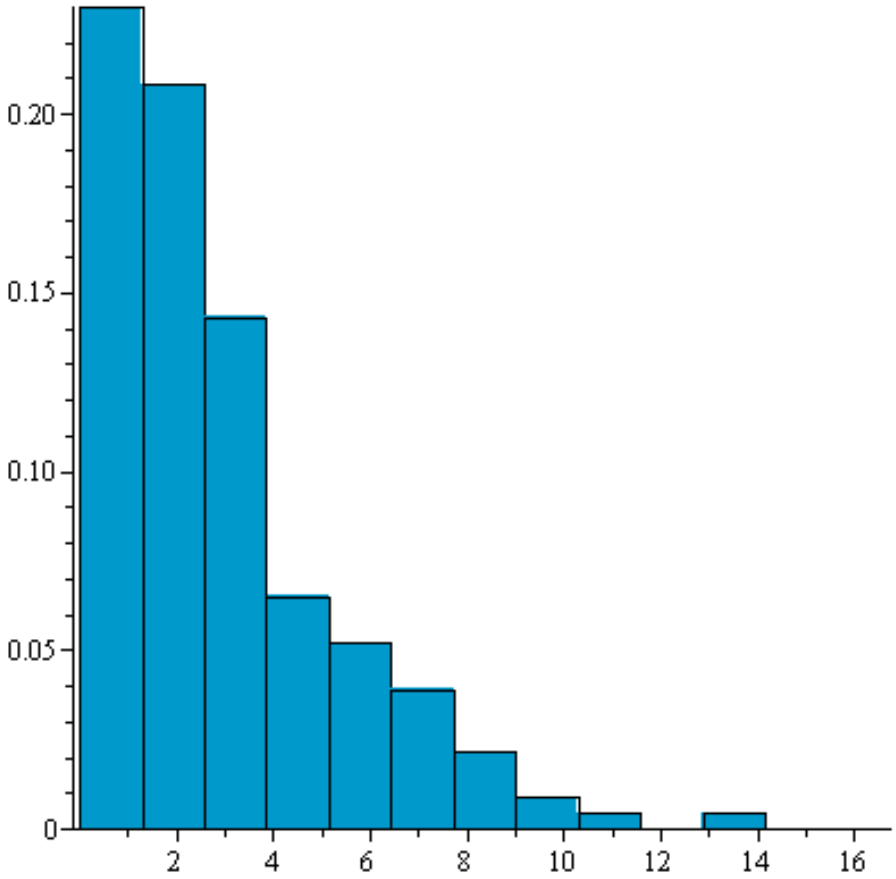

```
> W:=RandomVariable(Exponential(3));
```

$$W := \_R27$$

```
> Data3:=Sample(W,180);
```

$$Data3 := \begin{bmatrix} 1 .. 180\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
> Histogram(Data3,bincount=13);
Histogram Type:  default
Data Range:      .2563906058e-2 .. 16.74183353
Bin Width:       1.287636125
Number of Bins:  13
Frequency Scale: relative
```



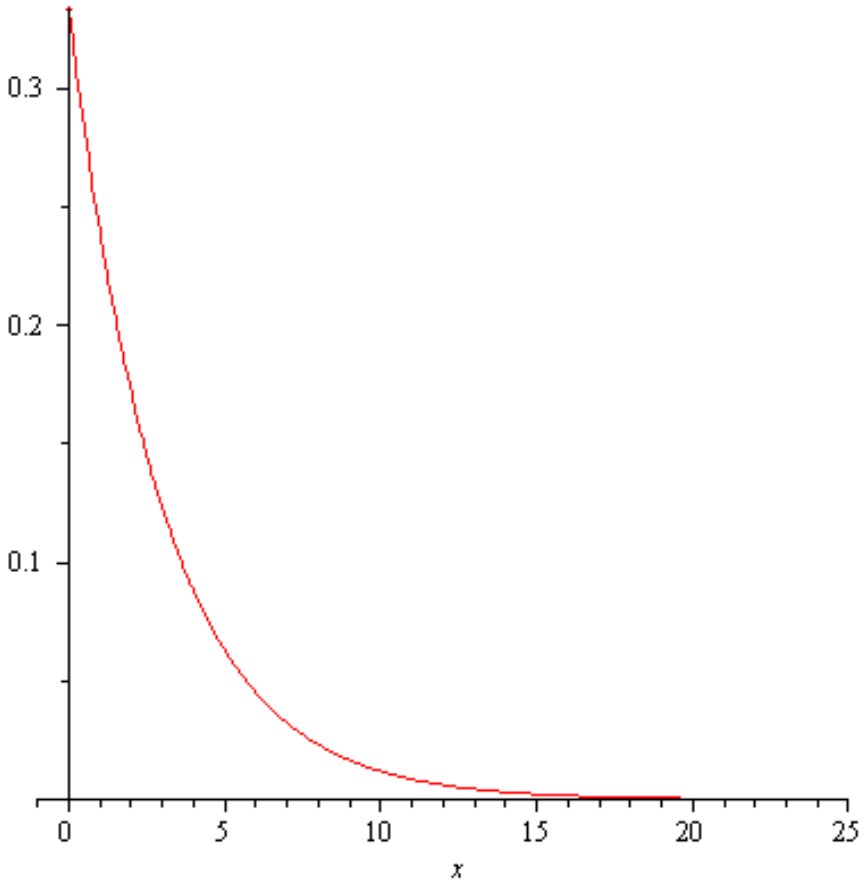

```
> PDF(Exponential(b),x);
```

$$\begin{cases} 0 & x < 0 \\ \dfrac{e^{-\frac{x}{b}}}{b} & otherwise \end{cases}$$

```
> PDF(W,x);
```

$$\begin{cases} 0 & x < 0 \\ \dfrac{1}{3} e^{-\frac{1}{3} x} & otherwise \end{cases}$$

```
> plot(,x=-1..25,discont=true);
```



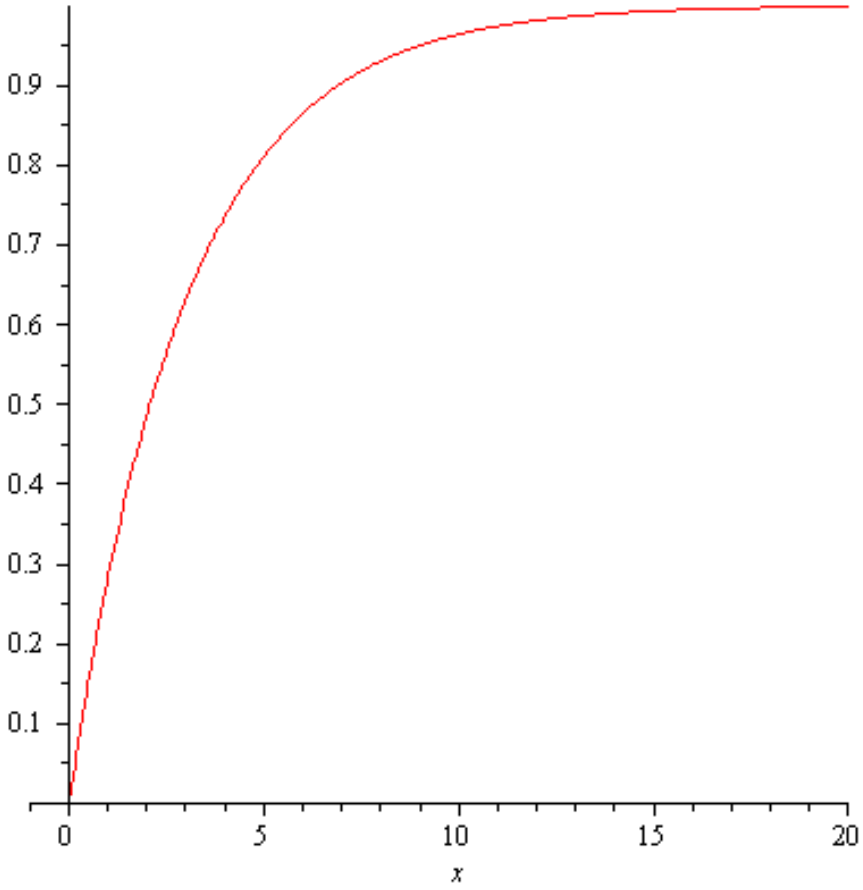

```
> CDF(Exponential(b),x);
```

$$\begin{cases} 0 & x < 0 \\ 1 - e^{-\frac{x}{b}} & otherwise \end{cases}$$

```
> CDF(W,x);
```

$$\begin{cases} 0 & x < 0 \\ 1 - e^{-\frac{1}{3}x} & otherwise \end{cases}$$

```
> plot(,x=-1..20);
```




```
> DataSummary(Data);
```

[*mean* = 3.46000000000000, *standarddeviation* = 1.72574094758797, *skewness* = 0.0498945141362920, *kurtosis* = 1.65186481722501, *minimum* = 1., *maximum* = 6., *cumulativeweight* = 100.]

```
> DataSummary(Data1);
```

[*mean* = 3.57000000000000, *standarddeviation* = 1.91662915992725, *skewness* = 0.408906967263225, *kurtosis* = 2.91340218197218, *minimum* = 0., *maximum* = 10., *cumulativeweight* = 200.]

```
> DataSummary(Data2);
```

[*mean* = -1.90781581262862, *standarddeviation* = 1.82232217533812, *skewness* = -0.0646595684560273, *kurtosis* = 2.58795005687518, *minimum* = -6.19082741067383, *maximum* = 2.51234399377509, *cumulativeweight* = 150.]

```
> DataSummary(Data3);
```

[*mean* = 2.93298878995111, *standarddeviation* = 2.68855033607880, *skewness* = 1.76398150589446, *kurtosis* = 7.39046442157756, *minimum* = 0.00256390605835740, *maximum* = 16.7418335331015, *cumulativeweight* = 180.]

```
> Mode(Data);
```

$$2.$$

```
> Mode(Data1);
```

$$3.998532290$$

```
> Mode(Data2);
```

$$-2.368368994$$

```
> Mode(Data3);
```

$$1.322774548$$

Interval spolehlivosti pro střední hodnotu a směrodatnou odchylku - 95%.

```
> X1:=RandomVariable(Normal(3,2.2));
```

$$X1 := \_R30$$

```
> Data4:=Sample(X1,300);
```

$$Data4 := \begin{bmatrix} 1 .. 300\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
> prum4:=Mean(Data4); sm_od:=StandardDeviation(Data4);
```

$$prum4 := 2.952284368$$

$$sm\_od := 2.30937510488223$$

```
> d4:=evalf(prum4-sm_od/sqrt(300)*Quantile(StudentT(299),0.975));
```

$$d4 := 2.68989669628626$$

```
> h4:=evalf(prum4+sm_od/sqrt(300)*Quantile(StudentT(299),0.975));
```

$$h4 := 3.21467203971374$$

```
> dd4:=sqrt(299*sm_od^2/Quantile(ChiSquare(299),0.975));
```

$$dd4 := 2.13818540770304$$

```
> hh4:=sqrt(299*sm_od^2/Quantile(ChiSquare(299),0.025));
```

$$hh4 := 2.51059289307529$$

Odhad střední hodnoty při házení regulérní kostkou.

```
> prum:=Mean(Data);
```

$$prum := 3.460000000$$

```
> st_od:=StandardDeviation(Data);
```

$$st\_od := 1.72574094758797$$

```
> d:=evalf(prum-sm_od/sqrt(100)*Quantile(Normal(0,1),0.975));
```

$$d := 3.00737079665752$$

```
> h:=evalf(prum+sm_od/sqrt(100)*Quantile(Normal(0,1),0.975));
```

$$h := 3.91262920334248$$

Vlastní rozdělení.

```
> piecewise(x<0,0,x<1,2/3,x<2,2/3*(2-x),0);
```

$$\begin{cases} 0 & x < 0 \\ \frac{2}{3} & x < 1 \\ \frac{4}{3} - \frac{2}{3}x & x < 2 \\ 0 & otherwise \end{cases}$$

```
> V:=Distribution(PDF=unapply(,x));
```

$$V := \textbf{module}(\,)$$
$$\quad \textbf{option}\ Distribution, Continuous;$$
$$\quad \textbf{export}\ Conditions, PDF, Type;$$
$$\textbf{end module}$$

```
> X2:=RandomVariable(V);
```

$$X2 := \_R37$$

```
> Mean(X2);
```

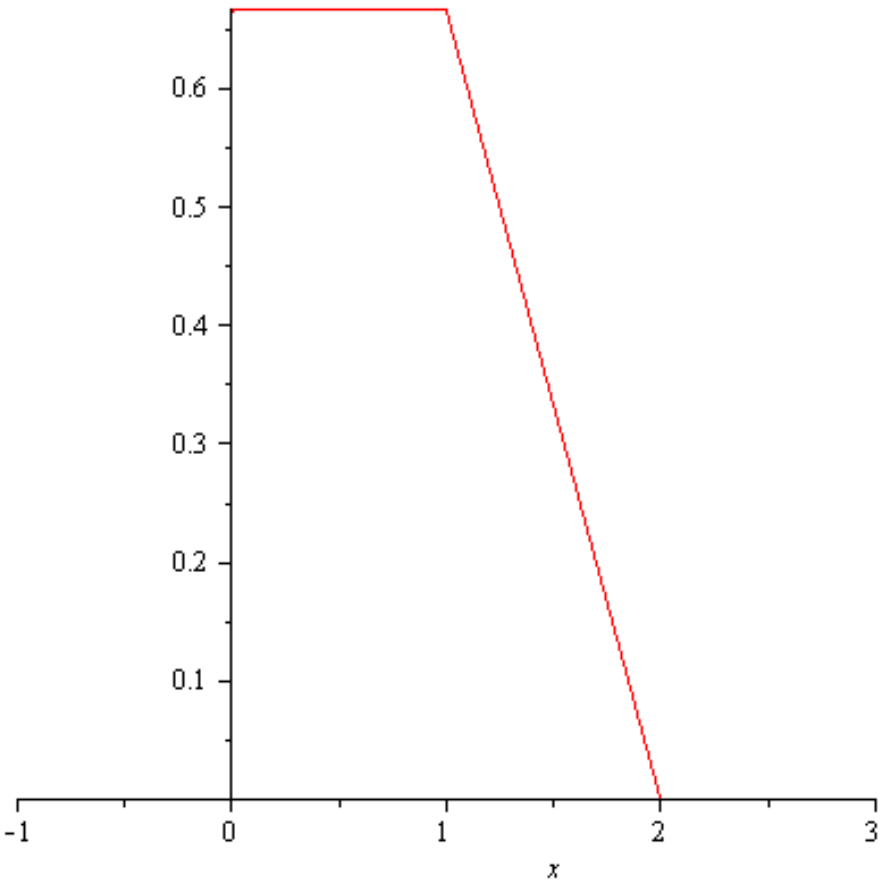
$$\frac{7}{9}$$

```
> StandardDeviation(X2);
```

$$\frac{1}{18}\sqrt{74}$$

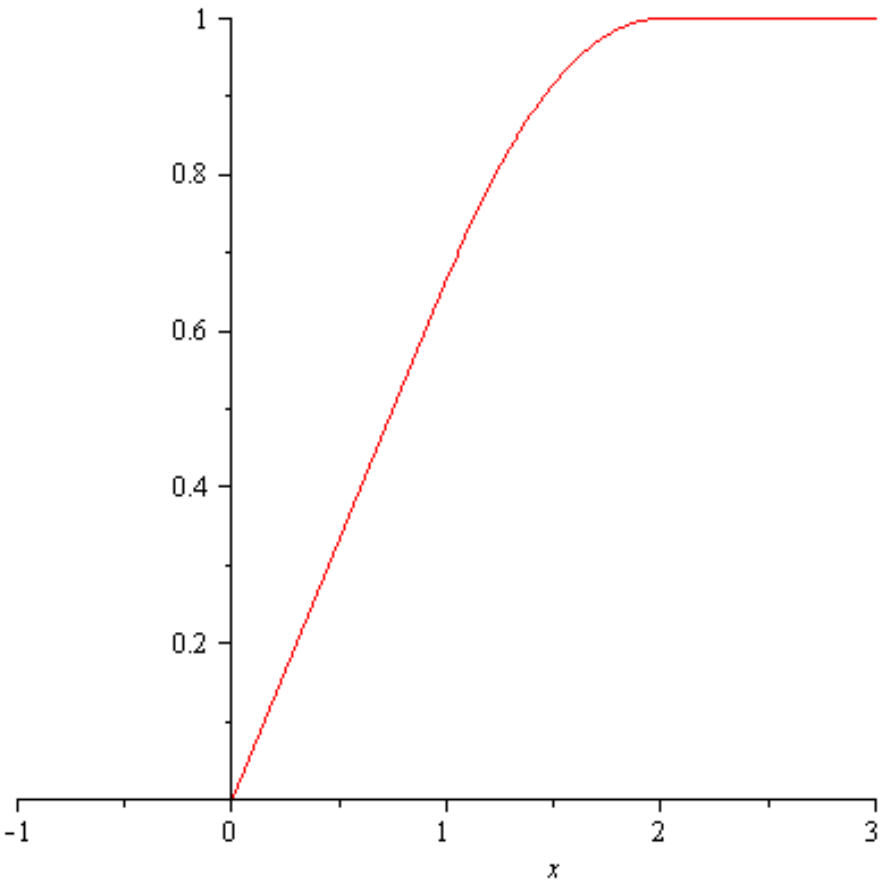
```
> PDF(X2,x); plot(PDF(X2,x),x=-1..3,discont=true);
```

$$\begin{cases} 0 & x < 0 \\ \frac{2}{3} & x < 1 \\ \frac{4}{3} - \frac{2}{3}x & x < 2 \\ 0 & otherwise \end{cases}$$




```
> CDF(X2,x); plot(CDF(X2,x),x=-1..3);
```

$$\begin{cases} 0 & x \le 0 \\ \frac{2}{3}x & x \le 1 \\ \frac{4}{3}x - \frac{1}{3}x^2 - \frac{1}{3} & x \le 2 \\ 1 & 2 < x \end{cases}$$




```
> Data5:=Sample(X2,50);
```

Error, (in Statistics:-Sample) could not evaluate derivative of PDF to floating point at 1.

```
> Sample(X+1,20);
```

Error, (in Statistics:-Sample) unable to use the adaptive rejection method for discrete random variates

```
> Mean(X2+1);
```

$$\frac{16}{9}$$

```
>
Sample(RandomVariable(Exponential(3))+RandomVariable(Poisson(1)),
20);
```

$$\begin{bmatrix} 1 .. 20\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
>
Sample(round(RandomVariable(Exponential(3)))+RandomVariable(Poiss
on(1)),20);
```

$$\begin{bmatrix} 1 .. 20\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
> Sample(Y^2+1,20);
```

$$\begin{bmatrix} 1 .. 20\ Vector_{row} \\ Data\ Type: float_8 \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix}$$

```
>
```