

(modifikaci úloha z minula)

PRIMAŘNÍ

$$z = 5x_1 + 15x_3 + 5x_4 \rightarrow \text{min}$$

$$x_1 \leq 50$$

$$2x_2 + x_3 \leq 80$$

$$x_1 + x_3 + 3x_4 \leq 95$$

$$x_{1,2,3,4} \geq 0$$

standardní tvar $\Rightarrow$ musíme
převést úlohu do standardní formy.

DUALNÍ: max, " $\leq$ ", w_1, w_2, w_3 , 4 omezení, transpozice koef.

$$f = 50w_1 + 80w_2 + 95w_3 \rightarrow \text{max}$$

$$w_1 + w_3 \leq 5$$

$$2w_2 \leq 0$$

$$w_2 + w_3 \leq 15$$

$$3w_3 \leq 5$$

$$w_{1,2,3} \geq 0$$

SLN $\rightarrow$ SLR

$$w_1 + w_3 + w_1' = 5$$

$$2w_2 + w_2' = 0$$

$$w_2 + w_3 + w_3' = 15$$

$$3w_3 + w_4' = 5$$

$$f - 50w_1 - 80w_2 - 95w_3 = 0 \text{ (max)}$$

KT $\checkmark$ $w_2 \geq 0 \checkmark$
 $\Rightarrow$ ST

$$w_{1,2,3} \geq 0, w_{1',2',3',4'} \geq 0$$

z.p.	w_1	w_2	w_3	w_1'	w_2'	w_3'	w_4'	b_i	$\mu = \frac{b_i}{a_{ik}}, a_{ik} > 0$
w_1'	1	0	1	1	0	0	0	5	5 $\tilde{w}_1 - \tilde{w}_4'$
w_2'	0	2	0	0	1	0	0	0	-
w_3'	0	1	1	0	0	1	0	15	15 $\tilde{w}_3 - \tilde{w}_4'$
w_4'	0	0	(3)	0	0	0	1	5	5/3 $\tilde{w}_4 : 3$
f	-50	-80	-95	0	0	0	0	0	max $\tilde{w}_5 + 95\tilde{w}_4'$
w_1'	1	0	0	1	0	0	-1/3	10/3	-
w_2'	0	(2)	0	0	1	0	0	0	0 $\tilde{w}_2 : 2$
w_3'	0	1	0	0	0	1	-1/3	40/3	40/3 $\tilde{w}_3 - \tilde{w}_2'$
w_3	0	0	1	0	0	0	1/3	5/3	-
f	-50	-80	0	0	0	0	95/3	475/3	max $\tilde{w}_5 + 40 \cdot \tilde{w}_2$

w_1'	1	0	0	1	0	0	$-\frac{1}{3}$	$\frac{10}{3}$	
w_2	0	1	0	0	$\frac{1}{2}$	0	0	0	-
w_3'	0	0	0	0	$-\frac{1}{2}$	1	$-\frac{1}{3}$	$\frac{40}{3}$	-
w_3	0	0	1	0	0	0	$\frac{1}{3}$	$\frac{5}{3}$	-
f	-50	0	0	0	40	0	$\frac{95}{3}$	$\frac{475}{3}$	max $25 + 50 \cdot \frac{5}{3}$
w_1	1	0	0	1	0	0	$-\frac{1}{3}$	$\frac{10}{3}$	
w_2	0	1	0	0	$\frac{1}{2}$	0	0	0	
w_3'	0	0	0	0	$-\frac{1}{2}$	1	$-\frac{1}{3}$	$\frac{40}{3}$	
w_3	0	0	1	0	0	0	$\frac{1}{3}$	$\frac{5}{3}$	
f	0	0	0	50	40	0	15	325	MAX
	x_1'	x_2'	x_3'	x_1	x_2	x_3	x_4		

$$\vec{w}_{opt} = \left(\frac{10}{3}, 0, \frac{5}{3}; 0, 0, \frac{40}{3}, 0 \right) : \\ f_{max} = 325$$

$$x_{min} = 325$$

$$\vec{x}_{opt} = (50, 40, 0, 15; 0, 0, 0)$$

$$z = 5x_1 + 15x_3 + 5x_4 \rightarrow \min$$

$$x_1 \geq 50$$

$$2x_2 + x_3 \geq 80$$

$$x_1 + x_3 + 3x_4 \geq 95$$

$$x_{1..4} \geq 0$$

$$\text{anul. z: } z - 5x_1 - 15x_3 - 5x_4 = 0 \text{ (min)}$$

dualität prüf. $\Rightarrow$ DSM

SLN $\rightarrow$ SLR

$$\begin{array}{rcll} x_1 & -x_1' & = 50 & \cdot (-1) \\ 2x_2 + x_3 & -x_2' & = 80 & \cdot (-1) \\ x_1 + x_3 + 3x_4 & -x_3' & = 95 & \cdot (-1) \\ z - 5x_1 - 15x_3 - 5x_4 & & = 0 \text{ (min)} & \\ x_{1..4} \geq 0, & x_{1..3}' \geq 0 & & \end{array}$$

$$\begin{array}{rcll} -x_1 & +x_1' & = -50 \\ -2x_2 - x_3 & +x_2' & = -80 \\ -x_1 - x_3 - 3x_4 & +x_3' & = -95 \\ z - 5x_1 - 15x_3 - 5x_4 & & = 0 \text{ (min)} \\ x_{1..4} \geq 0, & x_{1..3}' \geq 0 & & \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{KT} \\ \Rightarrow \text{fab. DSM} \end{array}$$

z.p.	x_1	x_2	x_3	x_4	x_1'	x_2'	x_3'	b_i	
x_1'	-1	0	0	0	1	0	0	-50	$\vec{x} = (0, 0, 0, 0, -50, -80, -95)$ FN
x_2'	0	-2	-1	0	0	1	0	-80	$\vec{w} = (0, 0, 0, 5, 0, 16, 5)$ DP
x_3'	-1	0	-1	-3	0	0	1	-95	$\tilde{z}_3 : (-3) \quad \min \left\{ \left \frac{-5}{-1} \right , \left \frac{-15}{-1} \right , \left \frac{-5}{-3} \right \right\} = \frac{5}{3}$
z	-5	0	-15	-5	0	0	0	0	$\min \quad \tilde{z}_4 + 5 \cdot \tilde{z}_3'$
x_1'	-1	0	0	0	1	0	0	-50	$\vec{x} = (0, 0, 0, \frac{95}{3}, -50, -80, 0)$ FN
x_2'	0	-2	-1	0	0	1	0	-80	$\tilde{z}_2 : (-2) \quad \vec{w} = (0, 0, \frac{95}{3}, \frac{10}{3}, 0, \frac{40}{3}, 0)$ DP
x_3'	$\frac{1}{3}$	0	$\frac{1}{3}$	1	0	0	$-\frac{1}{3}$	$\frac{95}{3}$	$\min \left\{ \left \frac{0}{-2} \right , \left \frac{\frac{40}{3}}{-1} \right \right\} = 0$
z	$-\frac{10}{3}$	0	$-\frac{40}{3}$	0	0	0	$-\frac{5}{3}$	$\frac{475}{3}$	$\min$
x_1'	-1	0	0	0	1	0	0	-50	$\tilde{z}_1 : (-1) \quad \vec{x} = (0, 40, 0, \frac{95}{3}, -50, 0, 0)$ FN
x_2'	0	1	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	40	$\vec{w} = (0, 0, \frac{95}{3}, \frac{10}{3}, 0, \frac{40}{3}, 0)$ DP
x_3'	$\frac{1}{3}$	0	$\frac{1}{3}$	1	0	0	$-\frac{1}{3}$	$\frac{95}{3}$	$\tilde{z}_3 + \frac{1}{3} \tilde{z}_1$
z	$-\frac{10}{3}$	0	$-\frac{40}{3}$	0	0	0	$-\frac{5}{3}$	$\frac{475}{3}$	$\min \quad \tilde{z}_4 - \frac{10}{3} \tilde{z}_1$

x_1	1	0	0	0	-1	0	0	50	
x_2	0	1	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	40	
x_3	0	0	$\frac{1}{3}$	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	15	
x_4	0	0	$-\frac{40}{3}$	0	$-\frac{10}{3}$	0	$-\frac{5}{3}$	325	MIN

$$\vec{x} = (50, 40, 0, 15; 0, 0, 0) \quad \underline{\underline{OPT}}$$

$$\vec{\mu} = (10/3, 0, 5/3; 0, 0, 40/3, 0) \quad \underline{\underline{OPT}}$$

$$DP \wedge TP \Rightarrow OPT$$

$$\underline{\underline{z_{min} = 325}}$$