MATICOVÉ HRY

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{m1} & \dots & \dots & a_{mn} \end{pmatrix} \quad \alpha$$

β

$\alpha \leq \beta$

$\alpha = \beta$ ⇒ sedlový bod a_{ij} ⇒ optimální čisté strat.

$A_0 = A_i$, $B_0 = B_j$

$v = a_{ij}$

$\alpha < \beta$ ⇒ ~~sedl. bod~~ ⇒ hledáme řešení v oboru smíš. strat.

$\vec{x}_0 = (x_1, \dots, x_m)$, $\vec{y}_0 = (y_1, \dots, y_n)$

MH 2×2 ⇒ vzorce

MH „větší" než 2×2 ⇒ redukce na 2×2

① princip dominování

Př)

$$A = \begin{pmatrix} -4 & 6 & 3 \\ -3 & -3 & 6 \\ 2 & -3 & 4 \end{pmatrix} \begin{matrix} -4 \\ -3 \\ -3 \end{matrix} \qquad \alpha = -3$$

$$\begin{matrix} 2 & 6 & 6 \end{matrix}$$

$\beta = 2$

$\alpha < \beta$ ⇒ hledáme opt. řeš. v oboru smíš. strat.

$\vec{x}_0 = (\ , 0, \ )$, $\vec{y}_0 = (\ , \ , 0)$

$$A' = \begin{pmatrix} -4 & 6 \\ \cancel{-3} & \cancel{-3} \\ 2 & -3 \end{pmatrix} \qquad A'' = \begin{pmatrix} -4 & 6 \\ 2 & -3 \end{pmatrix}$$

$\det A'' = -4 \cdot (-3) - 2 \cdot 6 = 0$

$a = -4 + (-3) - (2 + 6) = -15$

$x_1'' = \frac{-3-2}{-15} = \frac{1}{3}$ $\qquad x_2'' = \frac{-4-6}{-15} = \frac{2}{3}$

$y_1'' = \frac{-3-6}{-15} = \frac{3}{5}$ $\qquad y_2'' = \frac{2}{5}$ $\qquad v = \frac{0}{-15} = 0$

hra spravedlivá

$$\vec{x}_0 = \left(\frac{1}{3}, 0, \frac{2}{3}\right), \quad \vec{y}_0 = \left(\frac{3}{5}, \frac{2}{5}, 0\right)$$

② MH typu $2 \times n$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \end{pmatrix}$$

~~sedl. bod~~ $(\alpha < \beta)$

$\boxed{\vec{x}_0 = (x_1, x_2)}$ $\qquad \vec{y}_0 = (y_1, \dots, y_n)$

$M_j(x_1) = a_{1j} \cdot x_1 + a_{2j} \cdot x_2 = \left| x_2 = 1 - x_1 \right| = a_{1j} \cdot x_1 + a_{2j} \cdot (1 - x_1) = (a_{1j} - a_{2j}) \cdot x_1 + a_{2j}$

$M_1(x_1) = a_{11} \cdot x_1 + a_{21} \cdot x_2 = a_{11} x_1 + a_{21} \cdot (1 - x_1) = (a_{11} - a_{21}) \cdot x_1 + a_{21}$

odpovídá výhře A, jestliže B hraje B_1 a A hraje smíš.

$x_1 \in \langle 0, 1 \rangle$

$$\left. \begin{aligned} M(x_1) &= \min_j M_j(x_1) \\ M(x_1^0) &= \max_{x_1} M(x_1) \end{aligned} \right\} \text{MINIMAX}$$

$x_2^0 = 1 - x_1^0$

Př)

$$A = \begin{pmatrix} 2 & 3 & 11 \\ 7 & 5 & 2 \end{pmatrix} \begin{matrix} 2 \\ 2 \end{matrix} \qquad \alpha = 2$$

$$\begin{matrix} 7 & 5 & 11 \end{matrix}$$

$$\beta = 5$$

$\alpha < \beta \Rightarrow \boxed{\vec{x}_0 = (\ ,\ )}$

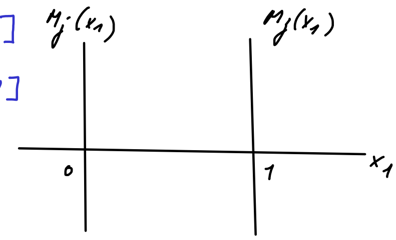
$\vec{y}_0 = (0,\ ,\ )$

na hru pohlížíme z pozice hráče A

$$M_1(x_1) = 2x_1 + 7x_2 = \left| \begin{matrix} x_1 + x_2 = 1 \\ x_2 = 1 - x_1 \end{matrix} \right| = 2x_1 + 7(1 - x_1) = -5x_1 + 7 \qquad [0,7];\ [1,2]$$

$$M_2(x_1) = 3x_1 + 5x_2 = 3x_1 + 5(1 - x_1) = -2x_1 + 5 \qquad [0,5];\ [1,3]$$

$$M_3(x_1) = 11x_1 + 2x_2 = 11x_1 + 2(1 - x_1) = 9x_1 + 2 \qquad [0,2];\ [1,11]$$



MINIMAX

$x_1 \in \langle 0, 1 \rangle$

$$M(x_1) = \min_j M_j(x_1)$$

$$M(x_1^0) = \max_{x_1} M(x_1)$$

hráč B určitě nevolí B_1

$$A' = \begin{pmatrix} 3 & 11 \\ 5 & 2 \end{pmatrix} \qquad \det A' = 6 - 55 = -49 \qquad a = 5 - 16 = -11$$

$$x_1' = \frac{2-5}{-11} = \frac{3}{11} \qquad x_2' = \frac{8}{11}$$

$$y_1' = \frac{2-11}{-11} = \frac{9}{11} \qquad y_2' = \frac{2}{11}$$

$$v = \frac{-49}{-11} = \frac{49}{11}$$

nespravedlivá (vzhl. A)

$$\vec{x}_0 = \left(\frac{3}{11}, \frac{8}{11}\right)$$

$$\vec{y}_0 = \left(0, \frac{9}{11}, \frac{2}{11}\right)$$

③ MH typu m×2

Př)

$$A = \begin{pmatrix} -2 & 1 \\ 1 & -5 \\ 0 & -1 \end{pmatrix} \begin{matrix} -2 \\ -5 \\ -1 \end{matrix} \qquad \alpha = -1$$

$$\begin{matrix} 1 & 1 \end{matrix}$$

$$\beta = 1$$

$\alpha < \beta$

$\vec{x}_0 = (\ , 0,\ )$

$\boxed{\vec{y}_0 = (\ ,\ )}$

na hru pohlížíme z pozice hráče B

$$N_1(y_1) = -2y_1 + y_2 = \left| \begin{matrix} y_1 + y_2 = 1 \\ y_2 = 1 - y_1 \end{matrix} \right| = -2y_1 + 1 - y_1 = -3y_1 + 1 \qquad [0,1],\ [1,-2]$$

$$N_2(y_1) = y_1 - 5 \cdot (1 - y_1) = 6y_1 - 5 \qquad [0,-5],\ [1,1]$$

$$N_3(y_1) = -(1 - y_1) = y_1 - 1 \qquad [0,-1],\ [1,0]$$

$y_1 \in \langle 0, 1 \rangle$

MAXIMIN

$$N(y_1) = \max_i N_i(y_1)$$

$$N(y_1^0) = \min_{y_1} N(y_1)$$

hráč A jistě nevolí A_2 ⇒ redukce MH 2x2

$$A' = \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix} \Rightarrow \text{vzorce} \ldots$$