Opakování středoškolské matematiky

1. Zjednodušte následující komplexní čísla (komplexní čísla zapište ve tvaru $a + b\mathrm{i}$).

   a) $-2 + 7\mathrm{i} + 5 - 2\mathrm{i}$, b) $2\mathrm{i}^2 + 2\mathrm{i}(1 - \mathrm{i})$, c) $\pi\left(\mathrm{i}^3 - 2\mathrm{i}^4\right)$,

   d) $\left(4 + \sqrt{2}\mathrm{i}\right)\left(\frac{\mathrm{i}}{2} - 1\right)$, e) $(-3 - \mathrm{i})^2 + 6(-3 - \mathrm{i}) + 10$, f) $(5 - 3\mathrm{i})(5 + 3\mathrm{i})$,

   g) $\frac{2 + 3\mathrm{i}}{7}$, h) $\frac{6}{\mathrm{i}}$, i) $\frac{2 + 3\mathrm{i}}{3 - \mathrm{i}}$, j) $\frac{2\mathrm{i}}{4 - \mathrm{i}} + \frac{\mathrm{i} - 2}{1 - 3\mathrm{i}}$.

2. Určete reálnou a imaginární část komplexních čísel $z_1 + z_2$, $z_1 z_2$ a z_1/z_2.

   a) $z_1 = 1 + 10\mathrm{i}, z_2 = 5$, b) $z_1 = 0, z_2 = 7 + 4\mathrm{i}$, c) $z_1 = 4 + \mathrm{i}, z_2 = -2\mathrm{i}$,
   d) $z_1 = 1, z_2 = 2 - 2\mathrm{i}$, e) $z_1 = 10 - 5\mathrm{i}, z_2 = 2\mathrm{i} - 1$, f) $z_1 = 2\mathrm{i} + 1 + \sqrt{2}\mathrm{i}, z_2 = 2(\mathrm{i} - 1) + 3$.

3. U komplexního čísla z určete $\operatorname{Re} z$, $\operatorname{Im} z$ a absolutní hodnotu $|z|$. Dále určete číslo komplexně sdružené k číslu z (tj. $\overline{z}$) a obě čísla z a $\overline{z}$ zobrazte v Gaussově rovině ($\sqrt{2} \doteq 1{,}4$; $\sqrt{3} \doteq 1{,}7$; $\sqrt{6} \doteq 2{,}4$).

   a) $z = 5\mathrm{i}$, b) $z = -3$, c) $z = \sqrt{3} + \mathrm{i}$,
   d) $z = 3\mathrm{i} - \sqrt{3}$, e) $z = -2 - 2\mathrm{i}$, f) $z = \sqrt{6} - \sqrt{2}\mathrm{i}$.

4. Zjednodušte následující výrazy.

   a) $|(2 + \mathrm{i})(3\mathrm{i} - 1)|$, b) $|(2 + \mathrm{i})(3\mathrm{i} - 1) + 1|$, c) $(2 + \mathrm{i})(2 - \mathrm{i}) - |2 + \mathrm{i}|^2 - (2 + \mathrm{i})^2$, d) $\left|\frac{2 + \mathrm{i}}{3\mathrm{i} - 1}\right|$.

5. Určete všechna komplexní řešení (kořeny) následujících kvadratických rovnic.

   a) $x^2 - 4 = 0$, b) $x^2 + 4 = 0$, c) $(x - 3)^2 + 4 = 0$, d) $x^2 - 4x + 8 = 0$,
   e) $x^2 + 6x + 10 = 0$, f) $x^2 - 15 = 0$, g) $x^2 + 15 = 0$, h) $x^2 - 4x + 6 = 0$,
   i) $x^2 + 8x + 28 = 0$, j) $4x^2 - 4x + 17 = 0$, k) $9x^2 + 12x + 13 = 0$, l) $2x^2 - 2x + 1 = 0$,
   m) $9x^2 + 6x + 2 = 0$, n) $4x^2 - 32x + 65 = 0$, o) $4x^2 + 24x + 45 = 0$, p) $4x^2 + 4x + 3 = 0$,
   q) $18x^2 - 12x + 11 = 0$.

Výsledky

1. a) $3 + 5\mathrm{i}$, b) $2\mathrm{i}$, c) $-2\pi - \pi\mathrm{i}$, d) $-\frac{8 + \sqrt{2}}{2} + (2 - \sqrt{2})\mathrm{i}$,
   e) 0, f) 34, g) $\frac{2}{7} + \frac{3}{7}\mathrm{i}$, h) $-6\mathrm{i}$, i) $\frac{3}{10} + \frac{11}{10}\mathrm{i}$, j) $-\frac{21}{34} - \frac{1}{34}\mathrm{i}$.

2. a) $\operatorname{Re}(z_1 + z_2) = 6,\ \operatorname{Im}(z_1 + z_2) = 10,\ \operatorname{Re}(z_1 z_2) = 5,\ \operatorname{Im}(z_1 z_2) = 50$,
   $\operatorname{Re}(z_1/z_2) = 1/5,\ \operatorname{Im}(z_1/z_2) = 2$,
   b) $\operatorname{Re}(z_1 + z_2) = 7,\ \operatorname{Im}(z_1 + z_2) = 4,\ \operatorname{Re}(z_1 z_2) = 0,\ \operatorname{Im}(z_1 z_2) = 0$,
   $\operatorname{Re}(z_1/z_2) = 0,\ \operatorname{Im}(z_1/z_2) = 0$
   c) $\operatorname{Re}(z_1 + z_2) = 4,\ \operatorname{Im}(z_1 + z_2) = -1,\ \operatorname{Re}(z_1 z_2) = 2,\ \operatorname{Im}(z_1 z_2) = -8$,
   $\operatorname{Re}(z_1/z_2) = -1/2,\ \operatorname{Im}(z_1/z_2) = 2$,
   d) $\operatorname{Re}(z_1 + z_2) = 3,\ \operatorname{Im}(z_1 + z_2) = -2,\ \operatorname{Re}(z_1 z_2) = 2,\ \operatorname{Im}(z_1 z_2) = -2$,
   $\operatorname{Re}(z_1/z_2) = 1/4,\ \operatorname{Im}(z_1/z_2) = 1/4$,
   e) $\operatorname{Re}(z_1 + z_2) = 9,\ \operatorname{Im}(z_1 + z_2) = -3,\ \operatorname{Re}(z_1 z_2) = 0,\ \operatorname{Im}(z_1 z_2) = 25$,
   $\operatorname{Re}(z_1/z_2) = -4,\ \operatorname{Im}(z_1/z_2) = -3$,
   f) $\operatorname{Re}(z_1 + z_2) = 2,\ \operatorname{Im}(z_1 + z_2) = 4 + \sqrt{2},\ \operatorname{Re}(z_1 z_2) = -3 - 2\sqrt{2},\ \operatorname{Im}(z_1 z_2) = 4 + \sqrt{2}$,
   $\operatorname{Re}(z_1/z_2) = 1 + 2\sqrt{2}/5,\ \operatorname{Im}(z_1/z_2) = \sqrt{2}/5$.

3. a) $\operatorname{Re} z = 0,\ \operatorname{Im} z = 5,\ |z| = 5, \overline{z} = -5\mathrm{i}$,
   b) $\operatorname{Re} z = -3,\ \operatorname{Im} z = 0,\ |z| = 3, \overline{z} = -3$,
   c) $\operatorname{Re} z = \sqrt{3},\ \operatorname{Im} z = 1,\ |z| = 2, \overline{z} = \sqrt{3} - \mathrm{i}$,
   d) $\operatorname{Re} z = -\sqrt{3},\ \operatorname{Im} z = 3,\ |z| = 2\sqrt{3}, \overline{z} = -\sqrt{3} - 3\mathrm{i}$,
   e) $\operatorname{Re} z = -2,\ \operatorname{Im} z = -2,\ |z| = 2\sqrt{2}, \overline{z} = -2 + 2\mathrm{i}$,
   f) $\operatorname{Re} z = \sqrt{6},\ \operatorname{Im} z = -\sqrt{2},\ |z| = 2\sqrt{2}, \overline{z} = \sqrt{6} + \sqrt{2}\mathrm{i}.$

4. a) $5\sqrt{2}$, b) $\sqrt{41}$, c) $-3 - 4\mathrm{i}$, d) $\dfrac{\sqrt{2}}{2}$.

5. a) ± 2, b) $\pm 2\mathrm{i}$, c) $3 \pm 2\mathrm{i}$, d) $2 \pm 2\mathrm{i}$, e) $-3 \pm \mathrm{i}$, f) $\pm\sqrt{15}$,
   g) $\pm\sqrt{15}\mathrm{i}$, h) $2 \pm \sqrt{2}\mathrm{i}$, i) $-4 \pm 2\sqrt{3}\mathrm{i}$, , j) $\dfrac{1}{2} \pm 2\mathrm{i}$, k) $-\dfrac{2}{3} \pm \mathrm{i}$, l) $\dfrac{1}{2} \pm \dfrac{1}{2}\mathrm{i}$,
   m) $-\dfrac{1}{3} \pm \dfrac{1}{3}\mathrm{i}$, n) $4 \pm \dfrac{1}{2}\mathrm{i}$, o) $-3 \pm \dfrac{3}{2}\mathrm{i}$, p) $-\dfrac{1}{2} \pm \dfrac{\sqrt{2}}{2}\mathrm{i}$, q) $\dfrac{1}{3} \pm \dfrac{\sqrt{2}}{2}\mathrm{i}.$